Estimating Monthly GDP in a General Kalman Filter Framework: Evidence from Switzerland[1,2]

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Abstract. *We estimate deseasonalized monthly series for Swiss Gross Domestic Product at constant prices of 1990 for the period 1980-1997. They are consistent with the quarterly figures estimated by the State Secretariat for Economic Affairs and obtained by including information contained in related series. We present a general approach using the Kalman filter technique nesting a great variety of interpolation setups. We evaluate competing models and provide a time series that can be used by other researchers.*

1. Introduction

For economic studies using quarterly data, a low number of observations can cause serious flaws in the quality of quantitative analysis. In vector autoregressions (VAR) with relatively short time series for example, many degrees of freedom are used up in the estimation, reducing drastically its power. Moreover, monthly frequency is sometimes implied by the assumptions of the model to be estimated, while only quarterly data are released[3].

Therefore, economists are sometimes forced to use variables that proxy GDP and that are available at a higher frequency. In many countries, a common proxy is industrial production (IP) which is often recorded at monthly frequency. In Switzerland, it is difficult

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[3] The official quarterly GDP figures for Switzerland are interpolated and published by the State Secretariat for Economic Affairs. Furthermore, an official annual GDP is calculated by the Federal Statistics Office producing the national income accounts. The quarterly estimates are then corrected and published again to match the official annual statistics.

to find such a monthly indicator for aggregate productive activity. The IP index is a series at a quarterly frequency, and other series like business surveys or filled orders can only be used as GDP proxies with some reservations. Hence, in cases where adequate proxies are not at hand, monthly estimates of GDP by interpolation can provide a solution to this problem[4].

Whether to replace a proxy variable by an interpolated one or not depends on the available data series and on the empirical economic model considered. The evaluation of the trade-off between potential benefits and disadvantages of both approaches is beyond the scope of this paper and is omitted. Our goal is to provide a monthly deseasonalized real GDP series for empirical research.

Chow and Lin (1971) were the first to present a coherent and easily applicable econometric approach that handles interpolation problems for stock and flow variables. Assuming a linear relation between the series of interest (series for which observations are missing, i.e. monthly GDP) and other data with more frequent recording (related series), they estimate a univariate regression equation. This multiple regression approach is flexible enough to take into account heteroscedasticity and low-order autocorrelation in the residuals. More recent studies make use of the Kalman filter (Harvey and Pierse (1984) and Bernanke, Gertler and Watson (1997)). This dynamic and flexible framework is capable of nesting more models than the Chow and Lin framework.

Here, we focus on econometric issues such as stationarity and cointegration in different Kalman filter configurations. Recent innovative techniques are analyzed theoretically and then evaluated empirically. We provide an overview of estimated monthly GDP series produced by various model setups. Then, we evaluate different combinations of methods and related series with the aim to get the most appropriate monthly GDP. For this task, several selection criteria as well as a simulated interpolation from annual to quarterly data are used.

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[4] Unlike other studies (e.g. Chow and Lin (1971)) our terminology does not distinguish between interpolation and distribution depending on whether stock and flow variables are used. The presented models exclusively serve for in-sample interpolations and not for out-of-sample predictions.

Before estimating the model, we evaluate competing related series. We identify the series containing the highest amount of information for the interpolation. The choice criteria for the related monthly series are based on the expenditure definition of GDP and on statistical properties of the comovement with GDP. However, the dearth of Swiss data at higher frequency limits severely the choice of these variables. Therefore, we consider other related series, for example foreign aggregate economic activity, as alternatives for interpolation. In fact, all related series that closely and robustly move together with quarterly GDP could be appropriate series helping to extract monthly GDP. With these related series available, it is then possible to estimate monthly GDP for Switzerland for 1980-1997 in different model setups[5].

The paper is organized as follows. It starts in section 2 with a short survey of the interpolation literature. In section 3, we briefly review the Kalman filter methodology and present the different interpolation models. In section 4, various related series are evaluated and described. We give an overlook of our results in section 5. We then evaluate the appropriateness of these interpolations. Section 6 concludes.

2. Related Literature

As Lanning (1986) illustrates, economists facing missing data have basically two different ways to solve that problem. A first approach is to estimate the missing data simultaneously with the economist's model parameters, thereby considering the missing observations as any other parameter. The second way is a two-step approach where in a first step the missing data, which could be independent from the economist's model, are interpolated. In a second step, the new augmented series are used to estimate the economist's model. Lanning found that the simultaneous approach yields estimates of the economist's model parameters that have a greater variance, and thus are less reliable, than the model

[5] We exclusively concentrate our investigation on the period 1980-1997 because these figures are compatible with the new national accounting system in Switzerland, the European System of Integrated Economic Accounts (ESA) 78. This standard was introduced in Switzerland in 1996, but the State Secretariat for Economic Affairs calculated quarterly GDP figures back to 1980. See Schwaller and Parnisari (1997) for a survey.

parameters estimated with complete data in the second stage. Based on these empirical findings, he suggests the use of the two-step approach. Related literature on the latter procedure can be subdivided in the following three classes.

First, the seminal approach for the use of the univariate multiple regression technique with related series was presented by Chow and Lin (1971, 1976) in a unified framework which allows treating the interpolation of stock and flow variables. This approach was able to overcome the problems faced by Friedman (1962) who treated stocks and flows in different ways. Specifically, they could deal with the requirement that if an observed flow value is distributed among the corresponding subintervals, the higher frequency estimates must add up to the original flow variable. Until now, this univariate regression approach has been widely used for interpolation due to its easier implementation than the state-space approach. This argument seems to more than just outweigh the potential advantages of more sophisticated procedures. An annual GDP is for example interpolated for Mexico by De Alba (1990). Schmidt (1986) gives a survey of this method, interpolating personal income of USA regions.

Second, Denton (1971), Fernandez (1981), and Litterman (1983) proposed a regression approach with related series that minimizes a weighted quadratic loss function on the difference between the series to be estimated and a linear combination of the observed related series. This model is related to the Chow and Lin regression and designed for the use of data in first difference. An illustration with Portuguese data is given in Pinheiro and Coimbra (1993).

Third, Bernanke, Gertler and Watson (1997) have used a state-space model to interpolate real GDP in the USA. Their approach is to first estimate monthly components of nominal GDP plus the GDP deflator and then to aggregate the individual estimates. They followed the methodology suggested by Harvey and Pierse (1984) who provide a general framework - state-space formulations for stock and flow variables, for stationary and nonstationary series, and with or without related series - to estimate missing observations in economic time series. Solving such state-space models requires the use of the Kalman filter. A Kalman filter interpolation is done for Canadian GDP by Guay, Milbourne, Otto and Smith (1990).

Hereafter, we present a state-space framework introduced by Harvey and Pierse (1984). This general formulation allows us to rewrite all three classes of models using related series as well as much simpler versions that do not make use of related series.

3. Models

3.1. Kalman Filter

A useful method for extracting signals is to write down a model linking the unobserved and observed variables in a state-space representation according to Kalman (1960, 1963). The multivariate Kalman filter is an algorithm for sequentially updating a linear projection on the vector of interest. We present various configurations of the state-space system in the next section on interpolation models.

The state-space representation is given by a system of two vector equations. First, the state or transition equation describes the dynamics of the state vector ($\boldsymbol{\xi}_t$) containing the unobserved variables we estimate. The second type of equation represents the observation or measurement equation linking the state vector to the vector containing the observed variables ($\mathbf{y}_t^+$). The equations of this system for $t = 1, \ldots, T$ where T is the number of monthly observations are the following:

$$\boldsymbol{\xi}_{t+1} = \mathbf{F}_t \boldsymbol{\xi}_t + \mathbf{C}_t' \mathbf{x}_{t+1} + \mathbf{R}_t \mathbf{u}_{t+1}, \qquad (1)$$

$$\mathbf{y}_t^+ = \mathbf{A}_t' \mathbf{x}_t^* + \mathbf{H}_t' \boldsymbol{\xi}_t + \mathbf{N}_t \mathbf{v}_t. \qquad (2)$$

In addition to the unobserved and the observed variables of interest, vector equations (1) and (2) contain the so-called related series ($\mathbf{x}_t$) and ($\mathbf{x}_t^*$) as exogenous variables in each equation. Both equations have multinormally distributed error terms:

$\begin{pmatrix} \mathbf{u}_t \\ \mathbf{v}_t \end{pmatrix} \sim N\left(\begin{pmatrix} \mathbf{0} \\ \mathbf{0} \end{pmatrix}, \begin{pmatrix} \mathbf{Q} & \mathbf{0} \\ \mathbf{0} & \mathbf{G} \end{pmatrix}\right)$. Premultiplied by matrices $\mathbf{R}_t$ and $\mathbf{N}_t$, these orthogonal disturbances transform into nonorthogonal residuals within each vector equation. The coefficients matrices $\mathbf{F}_t, \mathbf{C}'_t, \mathbf{R}_t, \mathbf{A}'_t, \mathbf{H}'_t, \mathbf{N}_t$, and the two variance-covariance matrices $\mathbf{Q}$ and $\mathbf{G}$ are estimated by maximizing the log-likelihood function of this system.

3.2. Interpolations Models

3.2.1. Overview

In this section, we adapt the general state-space representation (1) and (2) to our problem in different ways, specifically the inclusion of related series and assumed stochastic processes for monthly GDP. The interpolation framework[6] for $t = 1, \ldots, T$ is:

$$\boldsymbol{\xi}_{t+1} = \mathbf{F}\boldsymbol{\xi}_t + \mathbf{C}'\mathbf{x}_{t+1} + \mathbf{R}\mathbf{u}_{t+1}, \tag{3}$$

$$y_t^+ = \mathbf{a}'_t \mathbf{x}_t^* + \mathbf{h}'_t \boldsymbol{\xi}_t . \tag{4}$$

The state vector equation (3) describes the vector dynamics of the unobserved variable, monthly GDP y_t, stacked in the state vector $\boldsymbol{\xi}_t = \begin{pmatrix} y_t & y_{t-1} & y_{t-2} \end{pmatrix}'$. The exact formulation of this state vector equation is difficult, because there is no prior knowledge about the true process driving monthly GDP. In order to shed light on this issue, we compare results of various competing setups in section 5. We assume time-invariant coefficients for the matrices $\mathbf{F}, \mathbf{C}'$, and $\mathbf{R}$.

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[6] In all the models, quarterly GDP (y_t^+) is given each month, $y_1^+ = 0$, $y_2^+ = 0$, $y_3^+ =$ first quarterly value, $y_4^+ = 0$, $y_5^+ = 0$, $y_6^+ =$ second quarterly value, etc. Note that we observe $T/3$ quarterly values for T months to interpolate. Contrary to the usual convention we do not include zero observations when stacking monthly observations of quarterly values. The resulting column vector $\mathbf{y}^+$ has therefore the dimension $[T/3 \times 1]$.

Equation (4) relates the state vector to the observed quarterly GDP y_t^+. Following Harvey and Pierse (1984), this observation equation represents the constraint that the sum of three monthly GDP estimates within a quarter sum up to the quarterly data. The sum-up constraint is introduced by the coefficients vector $\mathbf{a}_t'$ and $\mathbf{h}_t'$ depending on the models presented in the following section. The identity character of this equality implies that it does not have to contain an error term.

All the specifications of the state-space models described hereafter correspond to different assumptions depending on whether related series ($\mathbf{x}_t$ and $\mathbf{x}_t^*$) are used or not and on the characteristics of the data to interpolate (stochastic process and stationarity). The properties of the data such as the order of integration and the assumed stochastic driving process of monthly GDP influence the representation of the state equation. Possible related series influence the setup of the state vector equation and the observation equation to in turn affect the coefficients contained in $\mathbf{C}'$ and $\mathbf{a}_t'$. We add related series in order to evaluate their statistical relevance. The selected assumptions are also guided by simplicity and technical considerations of the construction of the Kalman filter.

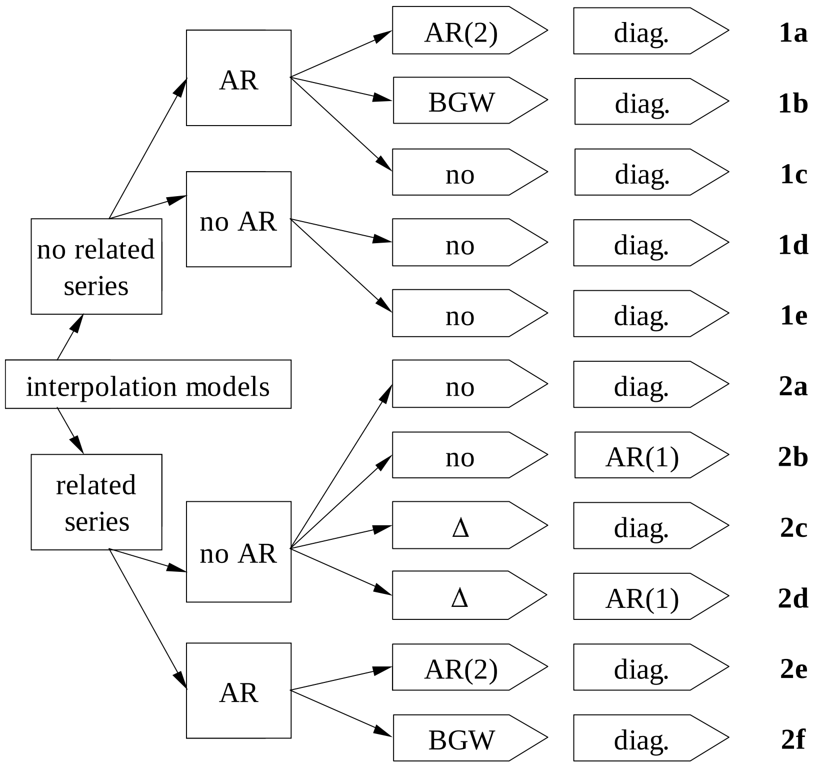
Hence, we focus on two broad classes of Kalman filter models summarized in figure 1.

The first class of models is designed without related series. We assume that there is enough information in the autocovariance function of the quarterly series and in the assumed low-order autoregressive (AR) process of monthly GDP. Moreover, we combine this assumption with alternative ways to treat nonstationary series (models 1a-c).

Contrasting with these AR models are two ‘naive’ models that neither follow an AR process nor include related series. However, it is not necessary to run the Kalman filter, because simple calculus produces the same results.

For each quarter, model 1d returns three equal monthly values, namely the third of the corresponding quarterly observation. Model 1e produces for each quarter three monthly GDP that follow a quarterly linear trend centered around the monthly mean of the quarter[7]. We take model 1e as a benchmark because of its intuitive setup.

Figure 1: *Overview of Interpolation Models*



First arrow column displays correction of nonstationarity. Second arrow column concerns the residuals form. Last column displays model numbers. AR(2) stands for an AR(1) process in first difference rewritten as an AR(2) in level; BGW means correction according to Bernanke, Gertler, and Watson (1997); Δ uses a first difference operator; in absence of correction done by the model, we

[7] Model 1d needs a constant term as an explanatory variable in order to calculate the third of the quarterly observation. Model 1e interpolates monthly observations linearly within a quarter, where we assume that we can split each quarter (except the first one) into an initial value y_{t-3} which is the last month of the previous quarter and a step d_t for $t = 4,5,\ldots,T$ according to the following equation: $(\varphi y_{t-3} + d_t) + (\varphi y_{t-3} + 2d_t) + (\varphi y_{t-3} + 3d_t) = y_t^+$. As quarterly GDP y_t^+, the step d_t is given each month, $d_4 = 0$, $d_5 = 0$, $d_6 =$ monthly step of second quarter, etc. φ is a scalar that takes on 1 for $t = 6,9,\ldots,T$ and 0 for $t = 4,5,7,\ldots,T-1$.

mention 'no'; 'diag.' indicates no autocorrelation in the residuals; AR(1) stands for residuals following an AR(1) process.

The second class of models is an extension of the first group in that it introduces related series in order to extract information for the interpolation of monthly GDP. Within this group, we distinguish between the assumptions that monthly GDP does not follow an autoregressive process (models 2a-d) and that it does (models 2e-f). We further enrich this second class of models with different ways to treat nonstationarity and with different assumptions about monthly residuals.

In the next paragraphs we show the various models 1a-c and 2a-f in detail.

3.2.2. Models without Related Series

Model 1a

In our first model, we assume that the first difference of monthly GDP Δy_t follows a stationary AR(1) process $\Delta y_t = \phi \Delta y_{t-1} + u_t$ in order to treat nonstationarity. Coefficient ϕ is constrained to lie inside the unit circle and u_t is an iid error term with distribution $N\left(0, \sigma_u^2\right)$. In order to find a starting value for the GDP series, it is imperative to write this AR(1) as an AR(2) of the series in level[8]:

$$y_t = (1+\phi) y_{t-1} - \phi y_{t-2} + u_t . \qquad (5)$$

This equation written in companion form, where $\xi_t = \begin{pmatrix} y_t & y_{t-1} & y_{t-2} \end{pmatrix}'$, yields the state equation (6) for $t = 1, \ldots, T$.

$$\begin{pmatrix} y_{t+1} \\ y_t \\ y_{t-1} \end{pmatrix} = \begin{pmatrix} 1+\phi & -\phi & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} y_t \\ y_{t-1} \\ y_{t-2} \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} u_{t+1} \\ u_t \\ u_{t-1} \end{pmatrix} \qquad (6)$$

[8] This 'transformation' yields the same likelihood and the same estimator for ϕ as the original equation. However, this form has the characteristic to produce a system with explosive eigenvalues.

Note that this formulation sets $\mathbf{C}' = \mathbf{0}$ in equation (3). The observation equation simply incorporates the sum-up constraint without related series. This implies $\mathbf{a}'_t = \mathbf{0}$. $\mathbf{h}'_t$ takes on two different values depending on the respective month. Namely, it is a row vector of ones when quarterly values are observable and it consists of zeros otherwise in order to cancel out the equation:

$$y_t^+ = \mathbf{h}'_t \boldsymbol{\xi}_t , \tag{7}$$

where $\mathbf{h}'_t = (0 \quad 0 \quad 0)$, for $t = 1,2,4,5,7,\ldots,T$, and where $\mathbf{h}'_t = (1 \quad 1 \quad 1)$, for $t = 3,6,9,\ldots,T$.

Model 1b

Recently, an interpolation method was suggested by Bernanke, Gertler and Watson (1997) who treat nonstationarity in an alternative way than model 1a. It consists in using GDP integrated of order one (I(1)) with a cointegrated series p_t such that we compute a new monthly stationary series[9] $y_t^s = y_t / p_t$. p_t is just a scaling variable such that y_t^s is nontrending. This approach relies on a calculated multiplicative cointegration that holds at both, monthly and quarterly frequencies. For the dynamic specification of y_t^s, now forming the elements of state vector $\boldsymbol{\xi}_t$, we assume AR(1) process $y_t^s = \phi y_{t-1}^s + u_t$. Compared to model 1a, y is replaced by y^s and the first row in $\mathbf{F}$ is $(\phi \quad 0 \quad 0)$. The observation equation also changes because we have to 'neutralize' the meaningless y_t^s series. Redefining vector $\mathbf{h}'_t$ restores the familiar sum-up constraint:

$$y_t^+ = \mathbf{h}'_t \boldsymbol{\xi}_t , \tag{8}$$

where $\mathbf{h}'_t = (0 \quad 0 \quad 0)$, for $t = 1,2,4,5,7,\ldots,T$, and where $\mathbf{h}'_t = (p_t \quad p_{t-1} \quad p_{t-2})$, for $t = 3,6,9,\ldots,T$.

[9] The ratio $y_t^s = y_t / p_t$ is chosen as a general framework in that it avoids to assume a particular cointegrating vector that we cannot estimate at the monthly level.

Model 1c

Finally, Bomhoff (1994) suggests to use the series in level arguing that the Kalman filter does not require the user to make a definite decision about the need for differencing the data. The Kalman filter offers automatic processing capacity for a wide range of nonstationary time series. Hence, we write down the law governing the process as if the series were stationary: $y_t = \phi y_{t-1} + u_t$. This model is similar to model 1a but with a first row of matrix $\mathbf{F}$ defined as $\begin{pmatrix}\phi & 0 & 0\end{pmatrix}$.

3.2.3. Models with Related Series and without AR Structure

The main criticism of models 1a-c is that they extract signals only from assumptions about the stochastic process of the original series without adding new information. We could speak about ‘fool-yourself’ models to generate monthly GDP. It is obvious that we are better off enriching the model with some economic content. For this purpose, we now include explanatory series that are related to the series to be interpolated.

In all the models, we may introduce the related series either in the measurement equation (4) for the generalized least squares (GLS) estimator (models 2a-d), or in the state equation (3) for the Kalman filter algorithm (models 2e-f).

Model 2a-b

Chow and Lin (1971, 1976) show how related series can be used to interpolate lower frequency data in order to get higher frequency data with a GLS estimator. They assume that monthly GDP y_t is described by a linear regression of y_t on l related series x_t, in matrix notation $\mathbf{y}_{GLS} = \mathbf{X}\boldsymbol{\beta} + \mathbf{u}_{GLS}$, where the variance-covariance of the error term is $\mathbf{V} = E\left(\mathbf{u}_{GLS}\mathbf{u}'_{GLS}\right)$[10]. They also

[10] In order to get identical coefficients $\boldsymbol{\beta}$ for the monthly and the quarterly regressions, the $[T \times 1]$ vector $\mathbf{y}_{GLS}$ of Chow and Lin contains figures that are three times larger than the monthly estimates of the Kalman filter setup.

assume the same relationship at the quarterly level: $\mathbf{y}^{+} = \mathbf{X}^{+}\boldsymbol{\beta} + \mathbf{u}^{+}$, where $\mathbf{X}^{+}$ is a matrix with quarterly average of three months of related series and $\mathbf{V}^{+}$ the variance-covariance matrix $E\left(\mathbf{u}^{+}\mathbf{u}^{+'}\right)$. $\mathbf{V}^{+}$ is thus a function of $\mathbf{V}$.

The Kalman filter configuration of the Chow and Lin model is:

$$\boldsymbol{\xi}_{t+1} = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \boldsymbol{\xi}_{t} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \mathbf{u}_{t+1}, \tag{9}$$

$$y_t^{+} = \mathbf{a}_t' \mathbf{x}_t^{*} + \mathbf{h}_t' \boldsymbol{\xi}_t, \tag{10}$$

where $\boldsymbol{\xi}_t = \begin{pmatrix} y_t - \mathbf{x}_t'\mathbf{c} \\ y_{t-1} - \mathbf{x}_{t-1}'\mathbf{c} \\ y_{t-2} - \mathbf{x}_{t-2}'\mathbf{c} \end{pmatrix}$ with $\mathbf{c} = \frac{1}{3}\boldsymbol{\beta}$, $\mathbf{x}_t^{*} = \sum_{j=t-2}^{t} \mathbf{x}_j$, $\mathbf{h}_t' = (0 \quad 0 \quad 0)$, and $\mathbf{a}_t' = \mathbf{0}$ for $t = 1,2,4,5,7,\ldots,T-1$, and $\mathbf{h}_t' = (1 \quad 1 \quad 1)$ and $\mathbf{a}_t' = \mathbf{c}'$ for $t = 3,6,9,\ldots,T$. As the related series and their coefficients are contained in the state vector equation, we set $\mathbf{C}' = \mathbf{0}$ in equation (3) and reintroduce them as $\mathbf{a}_t'\mathbf{x}_t^{*}$ in the observation equation.

Chow and Lin (1971, 1976) directly calculate a best linear unbiased estimator for the monthly series from the trace minimization of the covariance matrix $V[\hat{\mathbf{y}}_{GLS}]$, where $\hat{\mathbf{y}}_{GLS}$ denotes the $[T \times 1]$ vector of the monthly variables. Applying a GLS method, they avoid numerical optimization problems involved with the Kalman filter procedure[11]. The estimates of monthly GDP are

$$\hat{\mathbf{y}}_{GLS} = \mathbf{X}\hat{\boldsymbol{\beta}} + \boldsymbol{\Lambda}(\mathbf{V})\hat{\mathbf{u}}^{+}. \tag{11}$$

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[11] See the appendix for the comparison of log-likelihood functions for both Chow and Lin and the Kalman filter setups.

This special fitted value consist of two parts: a traditional fitted value $\mathbf{X}\widehat{\boldsymbol{\beta}}$ with the influence of related series and an interpolation-corrected residual term $\boldsymbol{\Lambda}(\mathbf{V})\widehat{\mathbf{u}}^{+}$.

$\widehat{\boldsymbol{\beta}}$ is a GLS estimator of the regression between quarterly GDP data ($\mathbf{y}^{+}$) and their 'quarterly' related series ($\mathbf{X}^{+}$):

$$\widehat{\boldsymbol{\beta}} = \left(\mathbf{X}^{+\prime} \mathbf{V}^{+^{-1}} \mathbf{X}^{+} \right)^{-1} \mathbf{X}^{+\prime} \mathbf{V}^{+^{-1}} \mathbf{y}^{+} . \tag{12}$$

The weighting matrix in this regression is the inverse of the variance-covariance matrix $\mathbf{V}^{+}$ of the quarterly residuals $\mathbf{u}^{+}$. Hence, the assumptions about $\mathbf{V}$ directly influence the distribution of $\widehat{\boldsymbol{\beta}}$ and the $[T \times T/3]$ matrix $\boldsymbol{\Lambda}$ for the dissemination of the quarterly residuals over the monthly estimated GDP. These quarterly residuals are crucial for interpolation, because the fitted monthly values $\mathbf{X}\widehat{\boldsymbol{\beta}}$ do not sum up to quarterly observations. To correct for this shortcoming, the residuals $\mathbf{u}^{+}$ must be 'redistributed' to the monthly GDP values according to the weighting matrix $\boldsymbol{\Lambda}$.

Models 2a and 2b differ in their assumptions about $\mathbf{V}$. In model 2a, we design the variance-covariance of the monthly residuals $\mathbf{V}$ simply as a diagonal matrix $\sigma^2_{u_{GLS}} \mathbf{I}_T$. It implies that $\mathbf{V}^{+}$, the variance-covariance matrix of quarterly residuals is equal to $\frac{\sigma^2_{u_{GLS}}}{3} \mathbf{I}_{T/3}$ and $\boldsymbol{\Lambda}$ is equal to $\mathbf{I}_{T/3} \otimes \mathbf{i}_3$.

However, a diagonal variance-covariance matrix $\mathbf{V}$ is rarely supported by the data. One way to account for this shortcoming is to allow for serial correlation in the error term. Hence, we assume for the model 2b that the error term follows an AR(1), $u_{GLS_t} = \varphi u_{GLS_{t-1}} + \varepsilon_t$ where ε_t is a white noise, yielding a variance-covariance matrix:

$$\mathbf{V} = \begin{pmatrix} 1 & \varphi & \varphi^2 & \dots & \varphi^{T-1} \\ \varphi & 1 & \varphi & & \\ \varphi^2 & \varphi & 1 & & \\ \vdots & & & \ddots & \\ \varphi^{T-1} & & & \dots & 1 \end{pmatrix} \frac{\sigma_\varepsilon^2}{1-\varphi^2}\,. \tag{13}$$

This specification obviously changes $\widehat{\boldsymbol{\beta}}$ and the redistribution matrix $\boldsymbol{\Lambda}$ which depends now on φ. The more φ tends to zero, the more the $\boldsymbol{\Lambda}$ matrix converges to $\mathbf{I}_{T/3} \otimes \mathbf{i}_3$. Hence, if the serial autocorrelation is significant, redistribution is less 'rigid' than in model 2a and the quarterly residuals are not only spread out over their corresponding months but also influence monthly GDP of surrounding quarters in a 'smoother' way.

Model 2c-d

A variation of models 2a-b, as suggested by Denton (1971) and Fernandez (1981), is to use first differences of a time series in the regression instead of levels in order to account for nonstationarity. In model 2c, we assume that the variance-covariance of the error term $\mathbf{V}$ is $\sigma^2_{u_{GLS}}\mathbf{I}_T$ and in model 2d, that the error terms follow an AR(1) with a variance-covariance matrix $\mathbf{V}$ equal to matrix (13). The weighting matrix to estimate $\widehat{\boldsymbol{\beta}}$ equals the inverse of the quarterly equivalent of $(\mathbf{D}'\mathbf{V}'\mathbf{D}')^{-1}$, where

$$\mathbf{D} = \begin{pmatrix} 1 & 0 & 0 & \dots \\ -1 & 1 & 0 & \\ 0 & -1 & 1 & \\ \vdots & & & \ddots \end{pmatrix}. \tag{14}$$

Note that compared to models 2a-b, the introduction of the first difference operator $\mathbf{D}$ affects matrix $\boldsymbol{\Lambda}$, and hence the redistribution of the quarterly residuals to the monthly GDP.

3.2.4. Models with Related Series and AR Structure

Model 2e

In addition to the introduction of related series explained in the previous class of models, we assume here in addition that monthly GDP is characterized by an autoregressive structure. The nonstationarity correction is similar as in model 1a. After the inclusion of related series to model 1a, the state equation becomes equation (6) plus the term $\mathbf{C}'\mathbf{x}_{t+1}$ where $\mathbf{x}_{t+1}$ includes l related series.

Model 2f

This model is similar to model 1b with added related series. As in model 2e, matrix $\mathbf{C}'$ characterizes their impact on monthly GDP.

4. Data

4.1. Signal Extraction from Related Series

A key factor in the present interpolation problem is the signal extraction from related series. Besides the assumption about the dynamics of GDP, related series data represent the main information source for interpolation. These data must fulfill two requirements.

First, they need to be correlated with the series to interpolate. The higher the systematic comovements with GDP are, the stronger is the signal to fill the gaps. If however, there is only a modest information content in the related series, this comes at the cost of a lot of noise introduced in the interpolated series. The choice of the related series is therefore crucial in order to successfully estimate a series at higher frequency.

Second, the related series need to be available in the desired higher frequency of the interpolated GDP. The fact that there are not many macroeconomic series available at monthly frequency imposes a strong restriction in Switzerland. This leads us to use also foreign variables that are correlated with the desired related series.

These two points require a thorough investigation for the task of choosing the correct related series. Amemiya (1980) suggests a joint strategy based on economic-theoretic considerations and on statistical evidence. Economic intuition can often indicate which related series to choose and what functional form they should have. Moreover, it is convenient to have a single statistical measure to choose related series that produce the 'best' result. These two aspects, intuitive approach and choice metrics, should be viewed as forming a single evaluation package rather than representing competitors. For the final choice of the related series, presented in detail in the following section, we jointly use both elements of the selection process.

4.2. Choice of Related Series

4.2.1. Economic Intuition

The most natural way to approach the series selection problem is to split up GDP into its expenditure components, private consumption (C), private domestic investments (I), government expenses (G), and net exports $(X-M)$:

$$Y = C + I + G + X - M \ . \qquad (15)$$

With the exception of exports and imports, none of these series is available at the higher frequency. Therefore, it is necessary to identify related series that proxy for the desired components.

An alternative to breaking GDP into its expenditure components is to benefit from the characteristics of Switzerland as a small open economy and the important comovement between domestic and foreign business cycles. Taking into consideration monthly foreign economic indicators allows us to choose the related series from a broader data set as Switzerland's closest trade partners have traditionally large statistical databases.

4.2.2. Statistical Evaluation

In this section, we describe the search for individual proxy variables within an economic model. Suppose, we identify a set of related data series $\mathbf{X}$ out of which variable x_k is unobservable. Furthermore, the variable y which is being interpolated depends linearly on $\mathbf{X}$.

$$y_t = \alpha_0 + \alpha_1 x_{1,t} + \alpha_2 x_{2,t} + ... + \alpha_k x_{k,t} + ... + u_t \qquad (16)$$

The goal is to choose the best observable proxy for x_k. In cases like this, an informal method often applied is replacing x_k with the variable z_k which yields the highest R^2 of all possible candidates in equation (16). Leamer (1983) shows that if the proxy variables are assumed to depend linearly on x_k and the error terms being normally iid, the best proxy is the one that produces the highest R^2. In the univariate regression $z_{i,t} = \delta_i x_{k,t} + \varepsilon_{i,t}$, the particular proxy z_i which yields the smallest variance $\sigma^2_{\varepsilon_i}$ can be defined as the best one. Leamer uses a likelihood ratio test to show the unambiguously negative relationship between the variance of the error term and R^2.

Another popular method which can be applied to a wider range of competing models than the one R^2 criterion above is the method of penalized likelihood. The best known examples in this class of criteria are the Akaike (1974) Information Criterion (AIC) and the Schwartz (1978) Information Criterion (SIC). In this class of criteria, a term that acts to punish additional coefficients is added to the negative of the likelihood function, and therefore, smaller values are preferable.

4.3. Data Description

Over a long time, Switzerland has stayed far behind other European countries in the development of economic statistical data. In 1996, as part of a reform program, national accounting was adapted to the European System of Integrated Economic

Accounts (ESA) 78. Thereafter, GDP was calculated differently. The Federal Statistics Office dated the series back to 1980 such that there is now a data sample of more than 18 years or 73 quarterly observations. The figures to be interpolated are deflated and deseasonalized.

The related series[12] in the national accounting approach have been identified as retail sales x^{rs} to proxy for private consumption and as the non-utilized construction loans to proxy for investment x^{nl}. These monthly available proxies have been selected based on the criteria described in the previous section. Furthermore, we include exports x^{X} and imports x^{M}. All the series are entered in levels, because the models transform the level vectors into the desired form. Government expenditure was dropped in the national accounting approach due to its low covariance with the business cycle. This would have introduced too much noise and moreover, there is no sensible proxy for it at monthly frequency.

As foreign series, we use a composite IP index of the five major trade partners of Switzerland x^{comip}, British IP x^{ukip}, and German IP x^{gip}. IP are the foreign monthly available series that move closest with the Swiss business cycle of all the related foreign series considered (results not reported). Prior to estimation, we have excluded several potential series based on economic arguments or on the statistical evaluation of the previous section. French IP, Italian IP, survey data by the Institute KOF for Business Cycle Research of the Swiss Federal Institute of Technology Zurich, labor market figures, exchange rates, and commodity prices were statistically eliminated. We have neither included variables that have proved to have predictive power for GDP such as the term spread because of unrealistic assumptions on the lead-lag relationship that would have been necessary. Figure 2 and table 1 give an overlook over the related series used in this paper.

During the 18 years of observations, the state of the Swiss economy can be roughly divided in two parts. Figure 2 clearly shows the phases of economic growth and prosperity in the 1980's

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[12] All the series, with the exception of real GDP given by the State Secretariat for Economic Affairs, are provided by Datastream.

and of stagnation in the 1990's. During its recession, Switzerland exhibited the lowest real GDP growth of all European countries[13].

Table 1 reports basic summary statistics of the quarterly and monthly series used for interpolation. Following the integration results from figure 2 and from augmented Dickey-Fuller (ADF) tests for all the variables (not reported), we find that all the series in levels are nonstationary. Hence, we report the results for growth rates. The ADF tests and the AR(1) regressions on the growth rates confirm that the level of the series is not stationary.

The different values of the contemporary cross-correlations also confirm the requirement of the comovements of the related series with quarterly GDP. Finally, these cross-correlations also show why we only consider contemporary relationships between the related series and the quarterly GDP. It is difficult to find robust leads and lags - the so-called stylized facts of the business cycles literature - between GDP and our proxy variables.

Table 1: Data Description

Descriptive Statistics					
	μ	σ	AR(1)	JB	ADF
gdp	1.33	2.95	0.25	0.05	-4.38*
x^{rs}	3.20	46.29	-0.65*	77.88*	-11.04*
x^{nl}	-0.97	21.39	0.25*	199.18*	-2.98*
x^{X}	4.09	49.03	-0.57*	53.17*	-7.91*
x^{M}	4.29	51.11	-0.61*	85.52*	-8.06*
x^{gip}	1.40	21.73	-0.43*	1370.58*	-6.26*
x^{ukip}	1.31	13.17	-0.22*	6.71*	-5.30*
x^{comip}	1.52	12.16	-0.32*	68.53*	-5.63*

[13] We decided not to take into account this structural break in the estimation of the Kalman parameters which could be done by the use of time-varying parameters or of dummy variables. As illustrated by our calculus models for example, the Kalman filter itself takes changing trends over time into consideration when computing monthly estimates.

Table 1: *Data Description (continued)*

Cross-correlations with gdp

	-3	-2	-1	0	1	2	3
x^{rs}	0.01	0.12	-0.01	0.09	0.13	0.12	0.02
x^{nl}	0.30	0.37	0.30	0.23	0.23	0.14	0.16
x^{X}	0.10	0.15	0.18	0.26	0.25	-0.02	-0.07
x^{M}	0.16	0.17	0.20	0.09	0.26	-0.05	0.03
x^{gip}	0.03	0.12	0.34	0.25	0.35	0.21	0.14
x^{ukip}	0.03	0.09	0.17	0.05	-0.01	-0.09	-0.17
x^{comip}	0.05	0.18	0.41	0.27	0.30	0.19	0.09

Annualized statistical figures are calculated for quarterly growth rates of GDP and for monthly growth rates for all other variables. *gdp* = Gross domestic product; x^{rs} = Value of retail sales; x^{nl} = Non-utilized construction loans; x^{X} = Exports volume; x^{M} = Imports volume; x^{gip} = IP in Germany; x^{ukip} = IP in UK; x^{comip} = Composite index of IP. All variables except x^{comip} are seasonally adjusted. μ = Mean; σ = Standard deviation; AR(1) = First-order autoregressive coefficient; JB = Jarque-Bera test; ADF = Augmented Dickey-Fuller test. Null hypotheses: i) first-order AR coefficient test, H_0: AR-coefficient = 0; ii) JB test, H_0: normal distribution; iii) ADF test, H_0: unit root. Rejection of the null hypothesis at the 1% significant level (*) and at the 5% significance level (**). Dynamic correlations with *gdp* are cross-correlations of lags and leads (between -3 and 3) of quarterly growth rate of related series with quarterly GDP growth rate. Source: Datastream and State Secretariat for Economic Affairs

We also perform a Johansen (1991) test in order to check for cointegration that is needed for the evaluation of the applicability of the Bernanke, Gertler and Watson (1997) approach. It is natural to assume that x^{rs} is moving along with GDP. We therefore test the quarterly proxy for cointegration. The test results reject the hypothesis of no cointegration at the 1% significance level. These results are displayed in table 2.

Table 2: *Cointegration Test of gdp and* x^{rs}

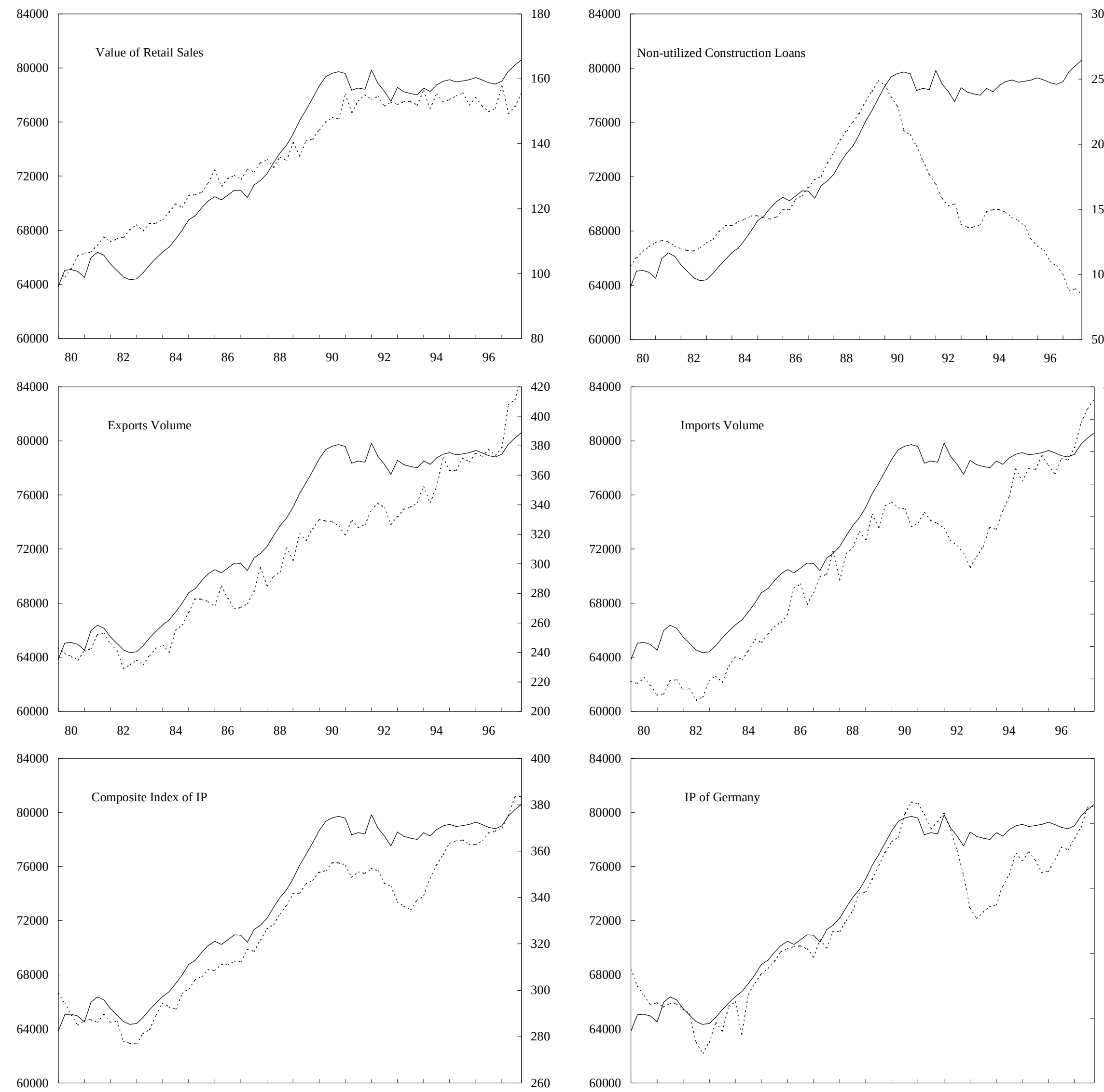
A	H_0	H_a	LR	B	H_0	H_a	LR
A1	0	2	25.11*	B1	0	1	23.88*
A2	1	2	1.73	B2	1	2	1.73

Cointegration tests are performed with quarterly data. *gdp* = Gross domestic product; x^{rs} = Value of retail sales. H_0 = Null hypothesis; H_a = Alternative hypothesis; for each hypothesis, given figure is number of cointegration relations; LR = Likelihood ratio statistic. Tests are run assuming linear trend in data and an intercept in the cointegrating equation and in the vector autoregression. Two lags are included. Test A, null hypothesis of h cointegrating relations against the alternative of no restrictions. LR is the weighted sum of the $(3-h)$-smallest eigenvalues. Test B, null hypothesis of h cointegrating relations against the alternative of $h+1$ relations. LR is the weighted h^{th} largest eigenvalue. Rejection of the null hypothesis at the 1% significant level (*) and at the 5% significance level (**).

We do not report the tests of other potentially cointegrated variables with an economic interpretation. All the tests reveal that only the quarterly GDP and x^{rs} are cointegrated. Hence, we use x^{rs} either as a related series or as the detrending series p in the Bernanke, Gertler and Watson (1997) framework[14]. Since we cannot directly test for multiplicative cointegration, an ADF test on the quarterly equivalent of $y^s = y/x^{rs}$ reveals that this ratio is stationary at the 1% significance level.

[14] To prevent the detrending series from introducing excessive volatility in the system, we take only the low frequency part of x^{rs} after Hodrick-Prescott filtering. The main objective of detrending GDP can still be maintained.

Figure 2: *gdp and Related Series*



GDP uses solid line in mio CHF; related series use dashed line. Non-utilized construction loans in mio CHF, other related series as index points. Source: Datastream and State Secretariat of Economic Affairs.

5. Results

5.1. Overview

The interpolation results are displayed in table 3. For each model, it contains statistical information about the estimated series for the period 1981-1997[15,16] namely, the related series, the information criterion, the log-likelihood, and key indicators for the annualized growth rate of the monthly interpolated GDP. Two mean-squared errors (MSE) for the evaluation of the models are given. The first one is between the level of the interpolated benchmark (model 1e) and the interpolated series of each model, respectively. The second one is the MSE between the observed quarterly GDP and a simulated quarterly interpolated GDP from annual data within the model in question in order to compare how the interpolation model would have performed at a frequency where models can be selected unambiguously based on an available data set.

Note, that table 3 is constructed in order to evaluate the models with respect to two basic directions. First, it is important to know whether the inclusion of related series (class 2) performs better than the 'fool-yourself' class 1. Second, we investigate the appropriate treatment of nonstationarity and analyze whether recent techniques perform better than traditional ones.

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[15] Due to initial oscillations using the Kalman filter, we discard twelve months of observations which otherwise would have influenced the results.

[16] Note that we interpolate GDP with information that is available ex post. This ensures that monthly values sum to the quarterly observations. In certain empirical studies, the monthly indicator should represent the information set of decision makers in the respective period. In Switzerland, quarterly GDP is published about 10 weeks after the reference quarter. In this case, we recommend to use a series not influenced by the sum-up constraint. The presented methods generate such a series as a by-product (not reported).

Table 3: *Interpolation Results*

Model	1, 1a	2, 1c	3, 2a	4, 2a
Series	-	-	x^{comip}	x^{nl}
AIC	8.05	10.08	14.66	17.31
log L	-562.56	-573.41	-636.59	-733.33
μ	1.32	1.33	1.20	1.25
σ	4.30	5.11	9.93	5.22
AR(1)	0.21*	0.06	-0.37*	-0.02
JB	308.06*	15.59*	12.09**	192.13*
ADF	-5.47*	-5.96*	-5.79*	-5.30*
MSE 1e	3307.38	4146.21	18659.04	7123.27
MSE AQ	196862.53	144095.19	206864.82	208184.95

Model	5, 2a	6, 2b	7, 2b	8, 2b
Series	x^{rs}, x^{nl}, x^{X}, x^{M}	x^{comip}	x^{nl}	x^{rs}, x^{nl}, x^{X}, x^{M}
AIC	13.06	15.89	17.32	13.34
log L	-575.38	-681.65	-733.97	-585.70
μ	1.27	1.29	1.30	1.41
σ	14.13	6.42	3.49	12.21
AR(1)	-0.53*	-0.14**	0.73*	-0.53*
JB	2.77	17.93*	16.83*	0.52
ADF	-6.40*	-5.53*	-4.20*	-6.85*
MSE 1e	32328.09	8884.46	4245.59	23601.90
MSE AQ	248244.08	129891.30	97166.50	221468.03

Model	9, 2c	10, 2c	11, 2c	12, 2d
Series	x^{comip}	x^{nl}	x^{rs}, x^{nl}, x^{X}, x^{M}	x^{comip}
AIC	15.63	14.42	14.38	15.27
log L	-677.97	-627.11	-623.52	-664.89
μ	1.30	1.30	1.34	1.27
σ	3.65	3.51	4.66	6.80
AR(1)	0.63*	0.72*	0.14**	-0.23*
JB	6.58**	15.93*	9.62*	13.45*
ADF	-4.30*	-4.17*	-5.57*	-4.91*
MSE 1e	4394.56	4287.16	5674.41	10947.86
MSE AQ	139871.77	102001.75	79448.95	120193.62

Table 3: *Interpolation Results (continued)*

Model	13, 2d	14, 2d	15, 2e	16, 2e
Series	x^{n}	x^{rs}, x^{nl}, x^{X}, x^{M}	x^{nl}	x^{rs}, x^{nl}, x^{X}, x^{M}
AIC	14.30	14.09	8.05	8.04
log L	-622.86	-614.83	-562.55	-562.55
μ	1.28	1.35	1.28	1.29
σ	4.69	7.96	4.30	4.58
AR(1)	0.10	-0.36*	0.22*	0.11
JB	332.63*	15.08*	306.50*	156.83*
ADF	-4.92*	-5.63*	-5.45*	-5.56*
MSE 1e	6696.91	11778.87	3282.52	3701.94
MSE AQ	151188.26	141821.70	147968.33	163693.19

gdp = Gross domestic product; x^{rs} = Value of retail sales; x^{nl} = Non-utilized construction loans; x^{X} = Exports volume; x^{M} = Imports volume; x^{comip} = Composite index of IP. All estimations include a constant, models 2c and 2d transform time trend to constant. Descriptive statistics are for annualized growth rates of the interpolated GDP for 1981-1997. log L = Value of log-likelihood function; μ = Mean; σ = Standard deviation; AR(1) = First-order autoregressive coefficient; JB = Jarque-Bera test; ADF = Augmented Dickey-Fuller test. Null hypotheses: i) first-order AR coefficient test, H_0: AR-coefficient = 0; ii) JB test, H_0: normal distribution; iii) ADF test, H_0: unit root. Rejection of the null hypothesis at the 1% significant level (*) and at the 5% significance level (**). MSE with 1e is for 1981-1997 and MSE Annual→Quarterly (AQ) is for 1982-1996.

5.2. Evaluation of Related Series

It is desirable that the interpolation not only relies on a purely econometric and mechanical procedure but also on economic intuition. Econometrically, the conclusion of whether to include related series or not is ambiguous. AIC and likelihood ratio tests show that introducing related series does not always enhance the performance of the interpolation as it involves costs of additional noise in the interpolated series. All the models generating too much volatility relative to the annualized standard deviation of the quarterly GDP estimates are not displayed in table 3 as they are economically not meaningful[17].

[17] We restrict ourselves to models that produce series with an annualized standard deviation lower than five times the variability of the growth rate of the

Related series could possibly break the regular pattern within a quarter, produced by all interpolation procedures without related series[18]. However, as shown in figure 3, series 16 for example is not able to break the pattern. The figure shows monthly estimates and the published quarterly GDP. The cyclical pattern within the quarter, illustrated in the bottom panels, is an average difference for the three months within the quarter between series 16 and the benchmark (1e) for growth and decline periods, respectively.

The deviations are significant for the first and the last observation within the quarter and lead us to reject the model for economic reasons. Moreover, we find that in all type 2e series the inclusion of related series, relative to model 1a, even exacerbates the pattern.

The suggestion of Bomhoff (1994) that the Kalman filter accounts for nonstationarity cannot be generalized for interpolation with AR structure in the state equation. Explosive eigenvalues, responsible for the pattern, are introduced in models 1a and 2e by construction. In model 1c, the pattern is implied by estimating a ϕ close to one. We eliminate the pattern by removing its source, the presumed inertia in GDP growth, and we use models 2a-d which assume no AR process.

Regarding the two sets of related series, one observes that in general, related series based on the open economy assumption introduce less volatility in the generated growth rates than the national accounting variables. For the related series x^{comip} and x^{nl} this relation is reversed[19]. However, including additional variables

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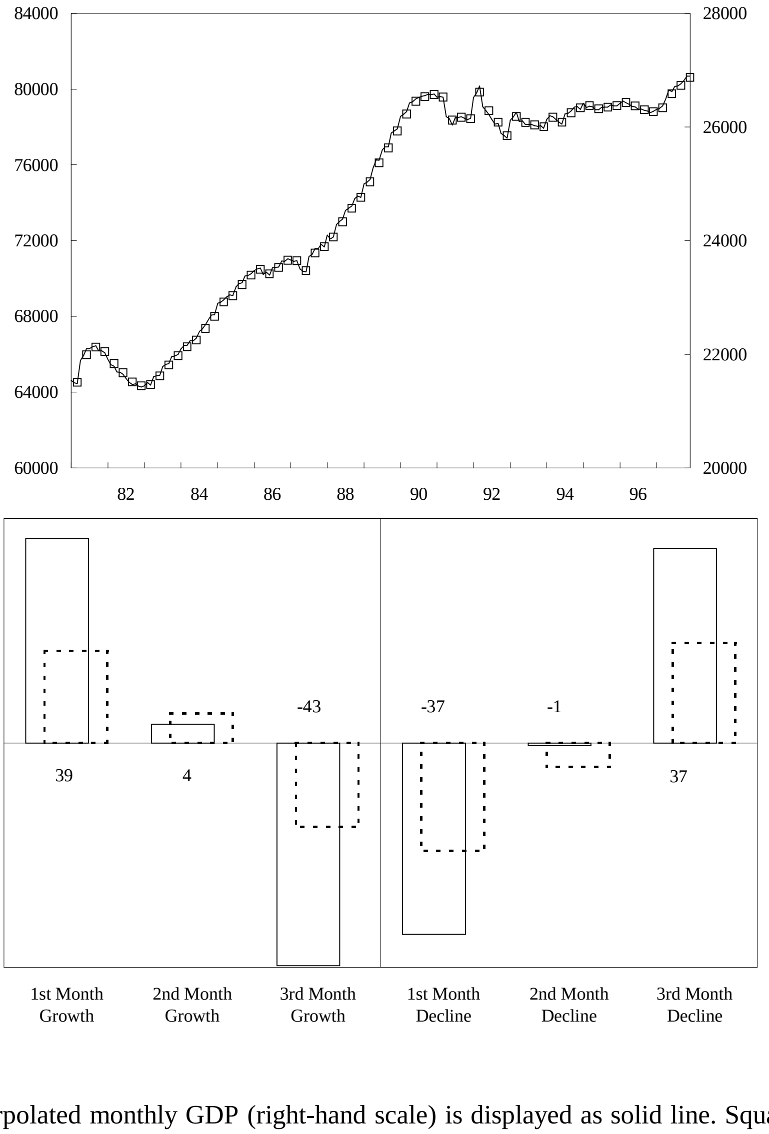
official quarterly GDP estimates (15%). Comparisons between monthly and quarterly values of industrial production growth in various countries show that the annualized values of monthly standard deviation are two to five times higher than quarterly ones which serve as a reference.

[18] The pattern is systematically convex or concave if the model has an AR structure, depending on growth state of the economy. Monthly GDP estimates produced by model 1e are linear and model 1d produces monthly estimates which equal one third of the corresponding quarterly GDP.

[19] Of all the series that could not be distinguished by statistical evaluation in section 4.2.2, x^{comip} is found to be the most useful related series of the open economy approach. Results using x^{gip} and x^{ukip} are therefore not reported in table 3.

in the national accounting approach increases the volatility considerably. To further investigate the characteristics of the most appropriate related series, note that within each model the log-likelihood values show that the national accounting approach is preferable even if not always significantly.

Figure 3: *Pattern of Series 16*



Interpolated monthly GDP (right-hand scale) is displayed as solid line. Squares represent quarterly GDP values. The bars in the bottom panels represent the average deviation of the monthly interpolated values from the benchmark during GDP growth (bottom left) and decline periods (bottom right). The dashed bars indicate the 5% critical values. All figures in mio CHF.

Another evaluation criterion is the MSE of a model series with respect to the benchmark 1e. The results indicate that in general adding related series increases the MSE reflecting an increase in volatility as the models deviate more from the smooth benchmark. As this criterion is rather soft and as there are models with the contrary effect, it does not seem suitable for model evaluation. Moreover, our benchmark is mainly founded on practical reasons and it proves not to be suitable as an objective measure for model evaluation.

5.3. Evaluation of Techniques

The comparison between different interpolation setups and the question whether modern setups perform better than traditional ones are closely linked to the treatment of stationarity. First of all, within the regression-based methods the correction for nonstationarity proposed by Denton (1971) and Fernandez (1981) (DF, model 2c) produces results that are qualitatively only slightly better than the classic Chow and Lin method using level series (CL, model 2a).

The effect of modelling AR(1) error terms in the CL-model (model 2b) and in the DF-model (model 2d) is not clear. In the CL-models, the likelihood falls while for the DF-models it increases, when AR(1) error terms are considered. The standard deviation of the generated series rises in the CL-models and behaves irregularly in the DF-setups.

Models constructed following Bernanke, Gertler and Watson (1997) are clearly worse than the ones reported in table 3, both in terms of cyclical regularity and volatility. This procedure neglects the fact that the Kalman filter already corrects the nonstationarity of the data.

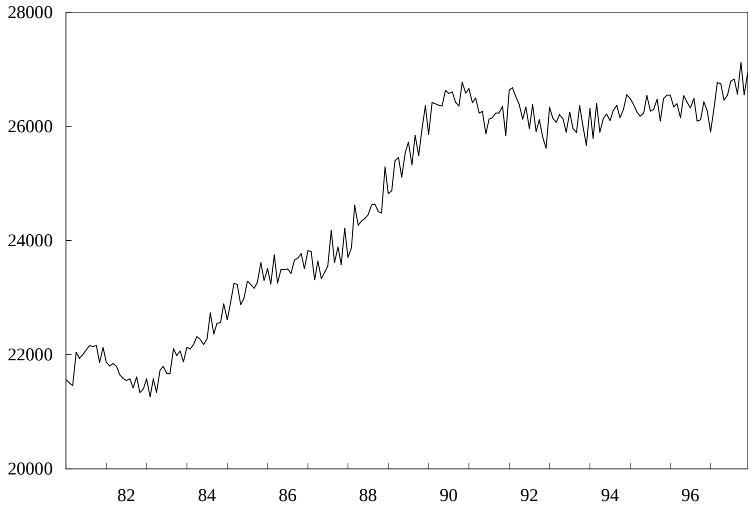
Generally, regression-based models yield good estimators while Kalman filter routines sometimes struggle with the numerical optimization. In case of model equivalence, we recommend for practical reasons the use of analytical solutions. However, due to its flexibility, the Kalman filter is able to model a much richer set of assumptions about the properties of monthly GDP while the

GLS approach utterly fails to model any stochastic behavior. This makes the Kalman filter an unavoidable tool when analyzing competing interpolation models.

5.4. A Monthly GDP Estimate

Based on this mixed evidence concerning the two directions, we recommend the setup and series 5 for further research. Due to the absence of AR structure, it does not display a regular pattern and the series exhibits moderate volatility as shown in figure 4.

Figure 4: *Monthly GDP Series*



Monthly GDP estimates in mio CHF. The corresponding figures are displayed in appendix D.

Can this extensive selection procedure be confirmed by first interpolating annual to quarterly data and then comparing the resulting quarterly series with the official GDP estimates? If yes, then we would have a very handy tool for the evaluation of competing interpolation models.

Of course, the underlying assumption that the best annual interpolation model is also the best quarterly one is strong, but if this criterion does well, it could be used as suggestive evidence in similar problems. Moreover, there is no reason to think that the frequency change has a fundamental impact on the performance of the models[20].

Surprisingly, the results show that it is not always the case that models with highest likelihood are the best interpolating models at the lower frequency. Within the GLS-based class just model 2c confirms our expectations. For all models with a pattern, applying this method makes no sense. Therefore, we conclude that this approach may be used as an indicator only but certainly not as a selection criterion.

6. Conclusion

The goal of this paper is to evaluate alternative interpolation models for Swiss GDP and to produce a monthly deseasonalized real GDP available for researchers and practitioners. We use a Kalman filter method which allows formulating a setup that nests a wide range of interpolation models in the literature. With respect to the nonstationarity and to the usefulness of related series, it is difficult to a priori present a clear-cut answer how these issues should be considered most suitably. Our results show that treating the nonstationarity problem with a second-order AR structure (models 1a, 2e) or with a detrending method (models 1b, 2f) is not appropriate for Swiss data. These two methods impose econometric characteristics on the produced data that cannot be carried further for an economic interpretation. The nonstationarity correction made by the filter itself (models 1c, 2a-b) seems to be sufficient.

Our results further show that in particular cases, related series can be useful. In these cases, the evaluation of series backed by economic intuition, is based on the comparison of the volatilities between the growth rates of the quarterly values and the computed monthly series, and some subsidiary indicators. We show that including related series does not systematically improve the results of the base case as this often generates quite volatile GDP estimates.

[20] Another way to apply this proposal would be to select the model with the best AIC for the interpolation from annual data and to see if the same model also produces the best AIC for the interpolation of monthly data from quarterly data.

Finally, the data does not seem to unambiguously confirm the expected long-run hypothesis between the interpolation at a monthly and at a quarterly level. A more rigorous econometric analysis would be needed if this comparison transgresses short-run considerations.

For the interpolation of Swiss GDP we suggest using an approach with the four related series exports, imports, retail sales, and non-utilized construction loans, the latter two of which are proxying for consumption and investment which are not monthly recorded.

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Appendix

A Kalman Filter Algorithm and Log-Likelihood Function

We show the iteration steps of the Kalman filter. We also give the log-likelihood function of our system. All our interpolation models are based on equations (3) and (4), $\boldsymbol{\xi}_{t+1} = \mathbf{F}\boldsymbol{\xi}_t + \mathbf{C}'\mathbf{x}_{t+1} + \mathbf{R}\mathbf{u}_{t+1}$, and $y_t^+ = \mathbf{a}_t'\mathbf{x}_t^* + \mathbf{h}_t'\boldsymbol{\xi}_t$. The Kalman filter iteration, update, prediction, and MSE steps at time t are given by the following loop. At time t assume that $y_0^+, \ldots, y_t^+$ are known. The related series $\mathbf{x}_t$ and $\mathbf{x}_t^*$ are known up to $t+1$. The predictions done at time $t-1$ for t are known: $\hat{\boldsymbol{\xi}}_{t|t-1}, \hat{y}_{t|t-1}^+$. The corresponding MSE are also known: $\mathbf{P}_{t|t-1} = MSE\left(\hat{\boldsymbol{\xi}}_{t|t-1}\right)$, and $MSE\left(\hat{y}_{t|t-1}^+\right)$.

<u>Update step</u>

$$\hat{\boldsymbol{\xi}}_{t|t} = \hat{\boldsymbol{\xi}}_{t|t-1} + \mathbf{P}_{t|t-1}\mathbf{h}_t\left(MSE\left(\hat{y}_{t|t-1}^+\right)\right)^{-1}\left(y_t^+ - \hat{y}_{t|t-1}^+\right)$$

$$\mathbf{P}_{t|t} = \mathbf{P}_{t|t-1} - \mathbf{P}_{t|t-1}\mathbf{h}_t\left(MSE\left(\hat{y}_{t|t-1}^+\right)\right)^{-1}\mathbf{h}_t'\mathbf{P}_{t|t-1}$$

<u>Prediction step</u>

$$\hat{\boldsymbol{\xi}}_{t+1|t} = \mathbf{F}\hat{\boldsymbol{\xi}}_{t|t} + \mathbf{C}'\mathbf{x}_{t+1}$$

$$\hat{y}_{t+1|t}^+ = \mathbf{a}_{t+1}'\mathbf{x}_{t+1}^* + \mathbf{h}_{t+1}'\hat{\boldsymbol{\xi}}_{t+1|t}$$

<u>MSE step</u>

$$MSE\left(\hat{\boldsymbol{\xi}}_{t+1|t}\right) = \mathbf{P}_{t+1|t} = \mathbf{F}\mathbf{P}_{t|t}\mathbf{F}' + \mathbf{R}\mathbf{Q}\mathbf{R}'$$

$$MSE\left(\hat{y}_{t+1|t}^+\right) = \mathbf{h}_{t+1}'\mathbf{P}_{t+1|t}\mathbf{h}_{t+1}$$

Log-likelihood function

Each observation y_t^+ is normally distributed:

$$y_t^+ \left| \left(y_0^+, \ldots, y_{t-1}^+, \mathbf{x}_1, \ldots, \mathbf{x}_t, \mathbf{x}_1^*, \ldots, \mathbf{x}_t^* \right) \sim N\left(\mathbf{a}_t' \mathbf{x}_t^* + \mathbf{h}_t' \widehat{\boldsymbol{\xi}}_{t|t-1}, \mathbf{h}_t' \mathbf{P}_{t|t-1} \mathbf{h}_t \right) \right.$$

The log-likelihood function for the whole sample is the following expression:

$$\sum_{t=1}^{\frac{T}{3}} \ln f\left(y_t^+\right) = -\frac{T}{6}\ln(2\pi) - \frac{1}{2}\sum_{t=1}^{\frac{T}{3}} \ln\left|\mathbf{h}_t'\mathbf{P}_{t|t-1}\mathbf{h}_t\right| - \frac{1}{2}\sum_{t=1}^{\frac{T}{3}} \frac{\left(y_t^+ - \mathbf{a}_t'\mathbf{x}_t^* - \mathbf{h}_t'\widehat{\boldsymbol{\xi}}_{t|t-1}\right)^2}{\mathbf{h}_t'\mathbf{P}_{t|t-1}\mathbf{h}_t}, \text{ or}$$

$$\sum_{t=1}^{\frac{T}{3}} \ln f\left(y_t^+\right) = -\frac{T}{6}\ln(2\pi) - \frac{1}{2}\sum_{t=1}^{\frac{T}{3}} \ln\left|\mathbf{h}_t'\left(\mathbf{F}\mathbf{P}_{t-1|t-1}\mathbf{F}' + \mathbf{R}\mathbf{Q}\mathbf{R}'\right)\mathbf{h}_t\right| - \frac{1}{2}\sum_{t=1}^{\frac{T}{3}} \frac{\left(y_t^+ - \mathbf{a}_t'\mathbf{x}_t^* - \mathbf{h}_t'\left(\mathbf{F}\widehat{\boldsymbol{\xi}}_{t-1|t-1} + \mathbf{C}'\mathbf{x}_t\right)\right)^2}{\mathbf{h}_t'\left(\mathbf{F}\mathbf{P}_{t-1|t-1}\mathbf{F}' + \mathbf{R}\mathbf{Q}\mathbf{R}'\right)\mathbf{h}_t}.$$

B Chow and Lin Regression[21]

We show hereafter the Chow and Lin regression model. Chow and Lin assume a true model for the monthly GDP explained by l related series given in matrix notation for the whole sample of T observations: $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{u}$ where $\mathbf{V} = E[\mathbf{u}\mathbf{u}']$ is the variance-covariance matrix of the error terms. With help of $[T/3 \times T]$ matrix $\mathbf{C}_D = \frac{1}{3}\left(\mathbf{I}_{T/3} \otimes \mathbf{i}_3'\right)$, they transform this true model to match the quarterly observed GDP. The quarterly vector can thus be expressed:

$$\mathbf{y}^+ = \mathbf{C}_D\mathbf{y} = \mathbf{C}_D\mathbf{X}\boldsymbol{\beta} + \mathbf{C}_D\mathbf{u} = \mathbf{X}^+\boldsymbol{\beta} + \mathbf{u}^+$$

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[21] We do not write GLS subscripts.

where $E\left[\mathbf{u}^{+}\mathbf{u}^{+'}\right] = \mathbf{V}^{+} = \mathbf{C}_D\mathbf{V}\mathbf{C}_D'$ and where $\mathbf{X}^{+}$ is a $[T/3 \times l]$ matrix with quarterly average of related series. Chow and Lin look then for a $[T \times T/3]$ matrix $\mathbf{A}$ that fills the gap between quarterly and estimated monthly data such that $\hat{\mathbf{y}} = \mathbf{A}\mathbf{y}^{+}$. In this search they impose an unbiased estimated monthly series $\hat{\mathbf{y}}$:

$$E[\hat{\mathbf{y}} - \mathbf{y}] = E\left[\mathbf{A}\left(\mathbf{X}^{+}\boldsymbol{\beta} + \mathbf{u}^{+}\right) - \mathbf{X}\boldsymbol{\beta} - \mathbf{u}\right] = E\left[\left(\mathbf{A}\mathbf{X}^{+} - \mathbf{X}\right)\boldsymbol{\beta}\right] = \mathbf{0}$$

implying that $\mathbf{A}\mathbf{X}^{+} - \mathbf{X} = \mathbf{0}$ and giving then an expression for the difference between the true monthly series and the estimated one: $\hat{\mathbf{y}} - \mathbf{y} = \mathbf{A}\mathbf{u}^{+} - \mathbf{u}$. To find the optimal matrix $\mathbf{A}$ they minimize, under the constraint of unbiasedness, the trace of the variance-covariance matrix $V[\hat{\mathbf{y}}]$, so minimizing the sum of all the variances corresponding to each 'observation'. The $V[\hat{\mathbf{y}}]$ is the following equation.

$$E\left[(\hat{\mathbf{y}} - \mathbf{y})^2\right] = \mathbf{A}E\left[\mathbf{u}^{+}\mathbf{u}^{+'}\right]\mathbf{A}' - \mathbf{A}E\left[\mathbf{u}^{+}\mathbf{u}'\right] - E\left[\mathbf{u}\mathbf{u}^{+'}\right]\mathbf{A}' + E[\mathbf{u}\mathbf{u}']$$

$$E\left[(\hat{\mathbf{y}} - \mathbf{y})^2\right] = \mathbf{A}\mathbf{V}^{+}\mathbf{A}' - \mathbf{A}\mathbf{V}^{+''} - \mathbf{V}''^{+}\mathbf{A}' + \mathbf{V}$$

They minimize with respect to $\mathbf{A}$ the following Lagrange function with help of a $[l \times T]$ Lagrange multiplier $\mathbf{M}'$:

$$L = \frac{1}{2}tr\left(\mathbf{A}\mathbf{V}^{+}\mathbf{A}' - \mathbf{A}\mathbf{V}^{+''} - \mathbf{V}''^{+}\mathbf{A}' + \mathbf{V}\right) - tr\left(\mathbf{M}'\left(\mathbf{A}\mathbf{X}^{+} - \mathbf{X}\right)\right)$$

$$L = \frac{1}{2}tr\left(\mathbf{A}\mathbf{V}^{+}\mathbf{A}'\right) - tr\left(\mathbf{A}\mathbf{V}^{+''}\right) + \frac{1}{2}tr(\mathbf{V}) - tr\left(\mathbf{X}^{+}\mathbf{M}'\mathbf{A}\right) + tr(\mathbf{M}'\mathbf{X})$$

yielding $\mathbf{A}$.

$$\mathbf{A} = \mathbf{X}\left(\mathbf{X}^{+'}\mathbf{V}^{+^{-1}}\mathbf{X}^{+}\right)^{-1}\mathbf{X}^{+'}\mathbf{V}^{+^{-1}}$$
$$+ \mathbf{V}''^{+}\mathbf{V}^{+^{-1}}\left(\mathbf{I}_n - \mathbf{X}^{+}\left(\mathbf{X}^{+'}\mathbf{V}^{+^{-1}}\mathbf{X}^{+}\right)^{-1}\mathbf{X}^{+'}\mathbf{V}^{+^{-1}}\right)$$

The $\hat{\mathbf{y}}$ is then given by the following fitted values: $\hat{\mathbf{y}} = \mathbf{A}\mathbf{y}^{+}$.

$$\hat{\mathbf{y}} = \mathbf{X}\left(\mathbf{X}^{+\prime}\mathbf{V}^{+^{-1}}\mathbf{X}^{+}\right)^{-1}\mathbf{X}^{+\prime}\mathbf{V}^{+^{-1}}\mathbf{y}^{+}$$
$$+\mathbf{V}''^{+}\mathbf{V}^{+^{-1}}\left(\mathbf{I}_{T/3} - \mathbf{X}^{+}\left(\mathbf{X}^{+\prime}\mathbf{V}^{+^{-1}}\mathbf{X}^{+}\right)^{-1}\mathbf{X}^{+\prime}\mathbf{V}^{+^{-1}}\right)\mathbf{y}^{+}$$
$$\hat{\mathbf{y}} = \mathbf{X}\hat{\boldsymbol{\beta}} + \left(\mathbf{V}''^{+}\mathbf{V}^{+^{-1}}\right)\hat{\mathbf{u}}^{+} = \mathbf{X}\hat{\boldsymbol{\beta}} + \boldsymbol{\Lambda}\hat{\mathbf{u}}^{+}$$

The monthly series is computed by the third of all the elements of the vector $\hat{\mathbf{y}}$.

C Comparison of Log-Likelihood Functions

In a slightly different notation, the Kalman filter presented in section 'Models with Related Series and without AR Structure' yields the same likelihood as Chow and Lin. This shows that the results are the same for both methods. We assume this structural equation $y_t = \mathbf{x}_t'\mathbf{c} + u_t$, for $t = 1, \ldots, T$, where u_t is iid and $E\left(u_t^2\right) = \sigma_u^2$.

We define state vector $\boldsymbol{\xi}_t = \begin{pmatrix} y_t - \mathbf{x}_t'\mathbf{c} \\ y_{t-1} - \mathbf{x}_{t-1}'\mathbf{c} \\ y_{t-2} - \mathbf{x}_{t-2}'\mathbf{c} \end{pmatrix}$, state equation

$\boldsymbol{\xi}_{t+1} = \mathbf{I}_3\begin{pmatrix} u_{t+1} \\ u_t \\ u_{t-1} \end{pmatrix}$, and measurement equation $y_t^{+} = \mathbf{a}_t'\mathbf{x}_t^{*} + \mathbf{h}_t'\boldsymbol{\xi}_t$,

where $\mathbf{x}_t^{*} = \sum_{j=t-2}^{t}\mathbf{x}_j$, $\mathbf{h}_t' = \begin{pmatrix}0 & 0 & 0\end{pmatrix}$ and $\mathbf{a}_t' = \mathbf{0}$ for $t = 1,2,4,5,7,\ldots,T-1$, and $\mathbf{h}_t' = \begin{pmatrix}1 & 1 & 1\end{pmatrix}$ and $\mathbf{a}_t' = \mathbf{c}'$ for

$t = 3,6,9,\ldots,T$. We further assume $\boldsymbol{\xi}_{t|t-1} = \begin{pmatrix} \hat{y}_{t|t-1} - \mathbf{x}_t'\mathbf{c} \\ \hat{y}_{t-1|t-1} - \mathbf{x}_{t-1}'\mathbf{c} \\ \hat{y}_{t-2|t-1} - \mathbf{x}_{t-2}'\mathbf{c} \end{pmatrix}$ and $\mathbf{P}_{t|t-1} = \begin{pmatrix} \sigma_u^2 & 0 & 0 \\ 0 & \sigma_u^2 & 0 \\ 0 & 0 & \sigma_u^2 \end{pmatrix}$.

Finally, the log-likelihood function for this Kalman filter is

$$\sum_{t=1}^{\frac{T}{3}} \ln f\left(y_t^+\right) = -\frac{T}{6}\ln(2\pi) - \frac{T}{6}\ln\left(3\sigma_u^2\right) - \frac{1}{6\sigma_u^2}\sum_{\tau=1}^{\frac{T}{3}}\left(y_{3\tau}^+ - \mathbf{a}_{3\tau}'\mathbf{x}_{3\tau}^* - \mathbf{h}_{3\tau}'\hat{\boldsymbol{\xi}}_{3\tau|3\tau-1}\right)^2, \text{ or}$$

$$\sum_{t=1}^{\frac{T}{3}} \ln f\left(y_t^+\right) = -\frac{T}{6}\ln(2\pi) - \frac{T}{6}\ln\left(3\sigma_u^2\right) - \frac{1}{6\sigma_u^2}\sum_{\tau=1}^{\frac{T}{3}}\left(y_{3\tau}^+ - \mathbf{c}'\left(\mathbf{x}_{3\tau} + \mathbf{x}_{3\tau-1} + \mathbf{x}_{3\tau-2}\right)\right)^2.$$

Chow and Lin assume the following regression $\mathbf{y}^+ = \mathbf{X}^+\boldsymbol{\beta} + \mathbf{u}^+$ or for each observation $y_t^+ = \boldsymbol{\beta}'\mathbf{x}_t^+ + u_t^+$ meaning that quarterly observations are $N\left(\boldsymbol{\beta}'\mathbf{x}_t^+, \sigma_{u^+}^2\right)$. The log-likelihood function for this Chow and Lin regression is

$$\sum_{t=1}^{\frac{T}{3}} \ln f\left(y_t^+\right) = -\frac{T}{6}\ln(2\pi) - \frac{T}{6}\ln\left(\sigma_{u^+}^2\right) - \frac{1}{2}\left(\sigma_{u^+}^2\right)^{-1}\sum_{\tau=1}^{\frac{T}{3}}\left(y_{3\tau}^+ - \boldsymbol{\beta}'\mathbf{x}_{3\tau}^+\right)^2.$$

This equation is equivalent to the Kalman filter log-likelihood with $\boldsymbol{\beta}' = 3\mathbf{c}'$, $\mathbf{x}_t^+ = \frac{1}{3}\mathbf{x}_t^*$, and the variance of quarterly observations $\sigma_{u^+}^2$ equals three times the variance of monthly observations σ_u^2.

D Monthly GDP Estimates

Jan-81	21563	Apr-83	21337	Jul-85	23286	Oct-87	23888
Feb-81	21503	May-83	21730	Aug-85	23230	Nov-87	23580
Mar-81	21461	Jun-83	21793	Sep-85	23165	Dec-87	24213
	64527		64860		69681		71681
Apr-81	22043	Jul-83	21670	Oct-85	23265	Jan-88	23706
May-81	21932	Aug-83	21666	Nov-85	23615	Feb-88	23872
Jun-81	22003	Sep-83	22104	Dec-85	23297	Mar-88	24615
	65978		65440		70177		72193
Jul-81	22082	Oct-83	21986	Jan-86	23504	Apr-88	24270
Aug-81	22157	Nov-83	22066	Feb-86	23235	May-88	24336
Sep-81	22141	Dec-83	21876	Mar-86	23751	Jun-88	24389
	66380		65928		70490		72995
Oct-81	22158	Jan-84	22130	Apr-86	23255	Jul-88	24449
Nov-81	21864	Feb-84	22096	May-86	23496	Aug-88	24622
Dec-81	22124	Mar-84	22178	Jun-86	23495	Sep-88	24639
	66146		66404		70246		73710
Jan-82	21867	Apr-84	22314	Jul-86	23501	Oct-88	24505
Feb-82	21802	May-84	22267	Aug-86	23419	Nov-88	24484
Mar-82	21843	Jun-84	22175	Sep-86	23661	Dec-88	25295
	65512		66756		70581		74284
Apr-82	21793	Jul-84	22275	Oct-86	23688	Jan-89	24823
May-82	21650	Aug-84	22734	Nov-86	23770	Feb-89	24877
Jun-82	21582	Sep-84	22359	Dec-86	23508	Mar-89	25400
	65025		67368		70966		75100
Jul-82	21547	Oct-84	22553	Jan-87	23824	Apr-89	25453
Aug-82	21577	Nov-84	22558	Feb-87	23811	May-89	25112
Sep-82	21417	Dec-84	22887	Mar-87	23310	Jun-89	25541
	64541		67998		70945		76106
Oct-82	21606	Jan-85	22616	Apr-87	23639	Jul-89	25723
Nov-82	21333	Feb-85	22898	May-87	23334	Aug-89	25329
Dec-82	21400	Mar-85	23248	Jun-87	23441	Sep-89	25842
	64339		68762		70414		76894
Jan-83	21575	Apr-85	23229	Jul-87	23553	Oct-89	25490
Feb-83	21259	May-85	22877	Aug-87	24176	Nov-89	25936
Mar-83	21575	Jun-85	22985	Sep-87	23613	Dec-89	26363
	64409		69091		71342		77789

Jan-90	25861	Jan-92	26642	Jan-94	26317	Jan-96	26551
Feb-90	26420	Feb-92	26679	Feb-94	25790	Feb-96	26342
Mar-90	26398	Mar-92	26515	Mar-94	26406	Mar-96	26399
	78679		79836		78513		79292
Apr-90	26368	Apr-92	26386	Apr-94	25901	Apr-96	26154
May-90	26359	May-92	26130	May-94	26142	May-96	26539
Jun-90	26634	Jun-92	26344	Jun-94	26216	Jun-96	26422
	79361		78860		78259		79115
Jul-90	26580	Jul-92	25961	Jul-94	26102	Jul-96	26327
Aug-90	26603	Aug-92	26383	Aug-94	26281	Aug-96	26494
Sep-90	26424	Sep-92	25909	Sep-94	26368	Sep-96	26095
	79607		78253		78751		78916
Oct-90	26361	Oct-92	26120	Oct-94	26155	Oct-96	26116
Nov-90	26774	Nov-92	25801	Nov-94	26303	Nov-96	26430
Dec-90	26582	Dec-92	25621	Dec-94	26557	Dec-96	26265
	79717		77542		79015		78811
Jan-91	26663	Jan-93	26336	Jan-95	26486	Jan-97	25906
Feb-91	26418	Feb-93	26146	Feb-95	26386	Feb-97	26328
Mar-91	26499	Mar-93	26070	Mar-95	26254	Mar-97	26771
	79580		78552		79126		79005
Apr-91	26233	Apr-93	26208	Apr-95	26179	Apr-97	26748
May-91	26261	May-93	26137	May-95	26238	May-97	26459
Jun-91	25869	Jun-93	25900	Jun-95	26544	Jun-97	26549
	78363		78245		78961		79756
Jul-91	26128	Jul-93	26251	Jul-95	26271	Jul-97	26792
Aug-91	26151	Aug-93	25966	Aug-95	26296	Aug-97	26834
Sep-91	26234	Sep-93	25892	Sep-95	26475	Sep-97	26566
	78513		78109		79042		80192
Oct-91	26237	Oct-93	26364	Oct-95	26098	Oct-97	27119
Nov-91	26355	Nov-93	25982	Nov-95	26486	Nov-97	26556
Dec-91	25840	Dec-93	25667	Dec-95	26547	Dec-97	26936
	78432		78013		79131		80611