GEAR, SPEED, REVOLUTIONS

This note links the end gear ratio with speed, engine revolutions, and wheel size.

Definitions

Time

One hour h has 60 minutes m.

$$1h = 60m \quad \text{or} \quad 1m = \frac{1}{60}h$$

Distance

One mile mi has 63360 inches in. One kilometer km has 100000 centimeters cm.

$$1mi = 63360in \quad \text{or} \quad 1in = \frac{1}{63360}mi$$

$$1km = 100000cm \quad \text{or} \quad 1cm = \frac{1}{100000}km$$

Wheel size

Wheel size is expressed by its radius d, diameter D ($D = 2d$), or circumference ($2\pi d = D\pi$), either in inches in or centimeters cm. The circumference is actually the travelled distance during one wheel turn wt.

$$1wt = 2\pi d$$

Speed

mph (kmh) expresses travelled miles (kilometers) per hour.

$$mph = \frac{mi}{1h} \quad \text{and} \quad kmh = \frac{km}{1h}$$

Engine revolutions

rpm expresses the engine revolutions r per minute.

$$rpm = \frac{r}{1m}$$

End gear ratio

The end gear ratio egr is the combination of differential and transmission gear ratios. egr gives the engine revolutions needed per wheel turn. It is expressed as $r:1$, r being the engine revolutions for one wheel turn.

$$egr = \frac{r}{1wt}$$

The driving gear (i.e. engine) turns r times for each turn of the driven gear (i.e. wheel). For $r > 1$, and with respect to the engine rotations, this results in a speed reduction and a torque increase. Put in other words, the output speed is r times slower than the input speed, but the output torque is r times more than the input torque.

The higher the ratio number, the lower the gear ratio. For example, a 10:1 gear ratio is higher numerically, but in terms of speed of the driven gear it is a lower gear ratio than 2:1. A low numbered ratio (i.e. a tall or high gear ratio) provides a high speed. A high numbered ratio (i.e. a short or low gear ratio) provides high torque and fast acceleration. Put in other words: the lower the number, the faster the car goes with the same number of engine revolutions; the higher the number, the better the car accelerates, but at the expense of high speed cruising.

Formulas

Putting together the definitions of egr and rpm, $egr \times 1wt = r$ and $rpm \times 1m = r$, and eliminating r, $egr \times 1wt = rpm \times 1m$, we get the number of wheel turns per minute (or per hour).

$$\frac{wt}{m} = \frac{rpm}{egr} \longrightarrow \frac{wt}{h} = 60\frac{rpm}{egr}$$

Formulas (imperial)

To get the travelled distance in inches (miles), the formula is adapted using the travelled distance during a wheel turn.

$$\frac{in}{h} = 2\pi d^{in}\left(60\frac{rpm}{egr}\right) \longrightarrow \frac{mi}{h} = 2\pi d^{in}\frac{1}{63360}\left(60\frac{rpm}{egr}\right)$$

The relation can be simplified for speed and engine revolutions.

$$\boxed{mph = \frac{\pi d^{in}}{528}\frac{rpm}{egr}} \qquad \boxed{rpm = mph \times egr\frac{528}{\pi d^{in}}}$$

A car drives fast with a high running engine, large wheels, and a tall or high gear ratio. Put in other words, to achieve a given speed, an engine has to run particularly fast in case of small wheels and/or a low or short gear ratio.

The relation can also be written using the diameter D and getting rid of π. For example for the speed

$$mph = \frac{\pi D^{in}}{2 \times 528}\frac{rpm}{egr} \approx \frac{D^{in}}{336}\frac{rpm}{egr}.$$

Formulas (metric)

By analogy, the equivalent equations are the following:

$$\frac{cm}{h} = 2\pi d^{cm}\left(60\frac{rpm}{egr}\right) \longrightarrow \frac{km}{h} = 2\pi d^{cm}\frac{1}{100000}\left(60\frac{rpm}{egr}\right).$$

The relation can be simplified for speed and engine revolutions.

$$\boxed{kmh = \frac{\pi d^{cm}}{833.\bar{3}}\frac{rpm}{egr}} \qquad \boxed{rpm = kmh \times egr\frac{833.\bar{3}}{\pi d^{cm}}}$$

Examples

Assume an egr=1 and engine rotations of rpm=1000. Moreover, the wheel has a circumference of two meters, i.e. 200cm ($d^{cm} \approx 32$). Therefore, due to the unity gear ratio, the vehicle travels 200000 centimeters per minute, or 2 kilometers per minute, or 120 kilometers per hour.

$$kmh = \frac{\pi d^{cm}}{833.\bar{3}}\frac{rpm}{egr} \longrightarrow kmh = \frac{\pi\frac{200}{2\pi}}{833.\bar{3}}\frac{1000}{1} = 120$$

In case of a lower gear ratio, i.e. $egr = 5$ or $egr = 10$, the vehicle drives more slowly.

$$kmh = \frac{\pi \frac{200}{2\pi}}{833.\bar{3}} \frac{1000}{5} = 24 \quad \text{and} \quad kmh = \frac{\pi \frac{200}{2\pi}}{833.\bar{3}} \frac{1000}{10} = 12$$

In case of smaller wheels, still with unity gear ratio, $d^{cm} = \frac{200}{4\pi} \approx 16$ or $d^{cm} = \frac{200}{8\pi} \approx 8$, the vehicle drives more slowly.

$$kmh = \frac{\pi \frac{200}{4\pi}}{833.\bar{3}} \frac{1000}{1} = 60 \quad \text{and} \quad kmh = \frac{\pi \frac{200}{8\pi}}{833.\bar{3}} \frac{1000}{1} = 30$$

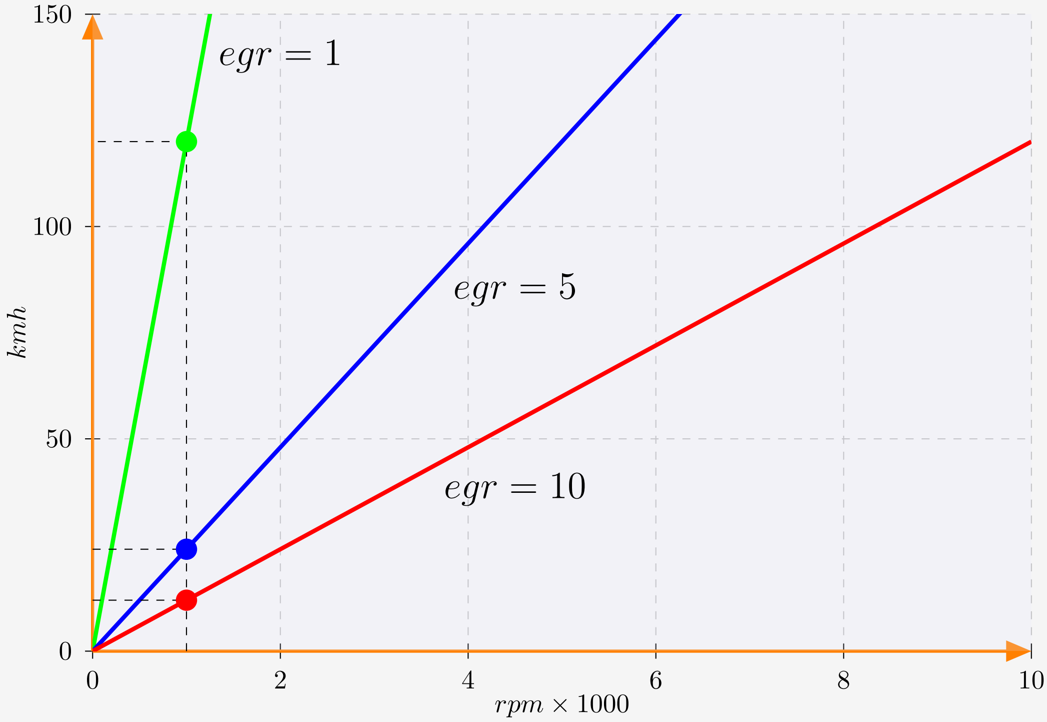

Figure 1: kmh as a function of rpm for different egr ($d^{cm} = \frac{200}{2\pi}$).

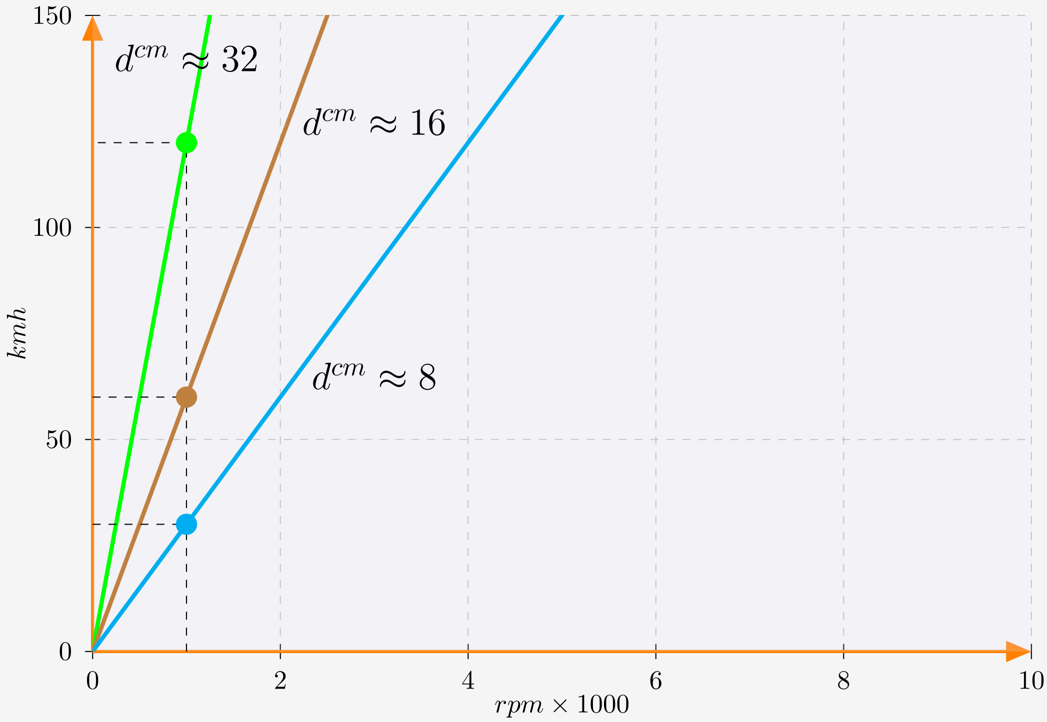

Figure 2: kmh as a function of rpm for different wheel size d^{cm} $(egr = 1)$.

(compiled September 26, 2024)