GEAR RATIO, TORQUE

This notes explains a few important principles about gears and torque.

Gears

Gears are toothed wheels which interlock to form simple machines. Considering a pair of gears, the driving gear (input gear) is the gear that is the source of rotation and engine torque. The driven gear (output gear) is the gear that is driven or rotated by the driving gear. The two gears, meshed together, transmit rotational motion and torque.

The driven gear can then rotate yet another gear. Thus, the second gear becomes the driving gear and the third gear is the driven gear. And so on.

Gears range in size, but the important number is how many teeth a gear has. When the teeth match well, there is only a small amount of friction. The two gears turn in opposite directions: one clockwise and the other counter-clockwise.

Gear ratio

A gear ratio is the ratio between the driving gear and the driven gear. Put in other words, the gear ratio is the ratio of the rotation number of a driving gear to the rotation number of a driven gear. A colon ':' is often used to represent a gear ratio: gr = rotations of driving gear : rotations of driven gear, the last figure being usually normalised to one.

A simple equation is used to find the ratio of the gearing system: number of teeth on the driven gear divided by number of teeth on the driving gear.

$$\frac{\text{number of teeth driven gear}}{\text{number of teeth driving gear}} = r \longrightarrow gr = r : 1$$

gr is expressed as $r : 1$, r being the revolutions of the first gear for one turn of the second gear. The driving gear turns r times for each turn of the driven gear. For $r > 1$, and with respect to the driving rotations, this results in a speed reduction and a torque increase. The output speed is r times slower than the input speed, but the output torque is r times more than the input torque.

The higher the ratio number, the lower the gear ratio. For example, a $10:1$ gear ratio is higher numerically, but in terms of speed of the driven gear it is a lower gear ratio than $2:1$. A low numbered ratio (i.e. a tall or high gear ratio) provides a high speed. A high numbered ratio (i.e. a short or low gear ratio) provides high torque and fast acceleration.

For $r > 1$, think of gear ratio as a divider on speed and a multiplier on torque. We gear up when we increase velocity and reduce torque. We gear down when we increase torque and reduce velocity. Gears convert torque to velocity, and vice-versa. The more velocity gained, the more torque sacrificed. If you get, for example, three times the original velocity, you reduce the resulting torque to one third. This conversion is symmetric. We can also convert velocity to torque at the same ratio. To sum up, if the speed is reduced, torque will increase by the same amount. If speed is increased, torque will decrease by the same amount.

Examples

Example 1

Plot 1 shows a pair of gears and their action may be compared to the action of two simple levers. The gear ratio of this example is:

$$\frac{19}{9} = 2.\bar{1} \quad \longrightarrow \quad gr = 2.\bar{1} : 1.$$

Put in other words, the small 9-tooth input gear must rotate $2.\bar{1}$ times to produce one revolution of the large 19-tooth output gear. Each gear has to rotate by the same number of teeth for them to correctly mesh. The radius of each gear is related – due to a good mesh – to the number of teeth on the gear. In this example the radius of the large gear is e.g. $84.\bar{4}mm$ and that of the small gear $40mm$. This also have a ratio of $2.\bar{1} = \frac{84.\bar{4}}{40}$.

The torque required at the road wheels of a vehicle is usually larger than the torque available at the engine flywheel. For example, the engine of a medium-sized car may develop a torque of $100Nm$ and require a torque of $1500Nm$ at the road wheels. This requires a torque multiplication of 15 times the engine torque. Thus, to operate the vehicle it is necessary to provide some means of multiplying engine torque. Use of the gears is the most commonly used method of torque multiplication on vehicles.

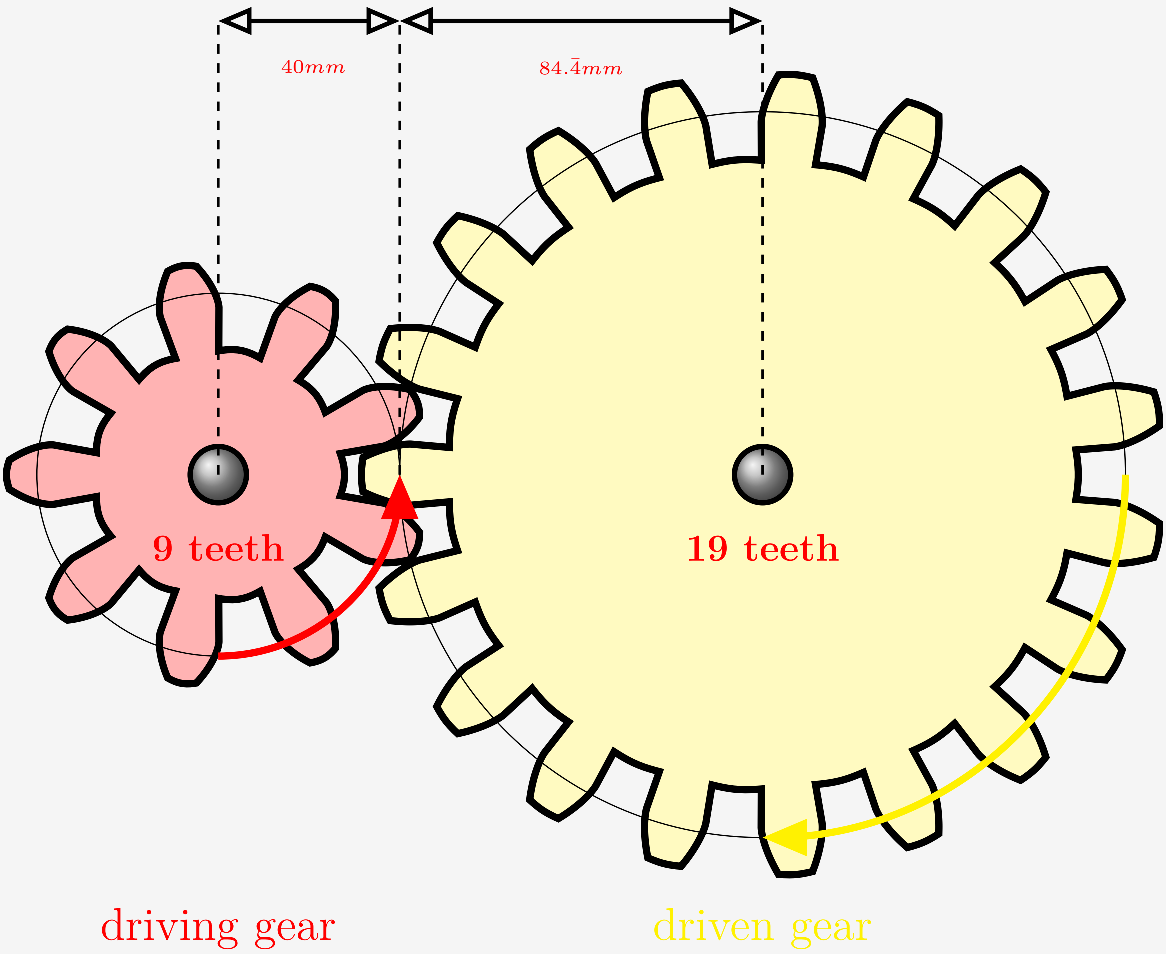

Figure 1: Example of two gears.

In this example, if we assume that the input gear has a force of $50N$, it carries a torque of $50N \times 40mm = 2000Nmm$, the torque on the large gear being $50N \times 84.\bar{4}mm = 4222.\bar{2}Nmm$.

Example 2

The principles are the same with e.g. three gears as plotted in Plot 2.

The different gear ratios are:

$$\frac{13}{5} = 2.6 > 1 \quad \longrightarrow \quad gr = 2.6 : 1$$

and

$$\frac{11}{13} \approx 0.85 < 1 \quad \longrightarrow \quad gr \approx 0.85 : 1,$$

and the end (final) gear ratio is the multiplication of both ratios, i.e.

$$2.6 \times 0.85 \approx 2.2.$$

In this example, it corresponds to the gear ratio between the first and last gear,

$$\frac{11}{5} = 2.2 \longrightarrow gr = 2.2 : 1$$

the middle gear being here only to inverse the rotation of the last gear. It size does not matter.

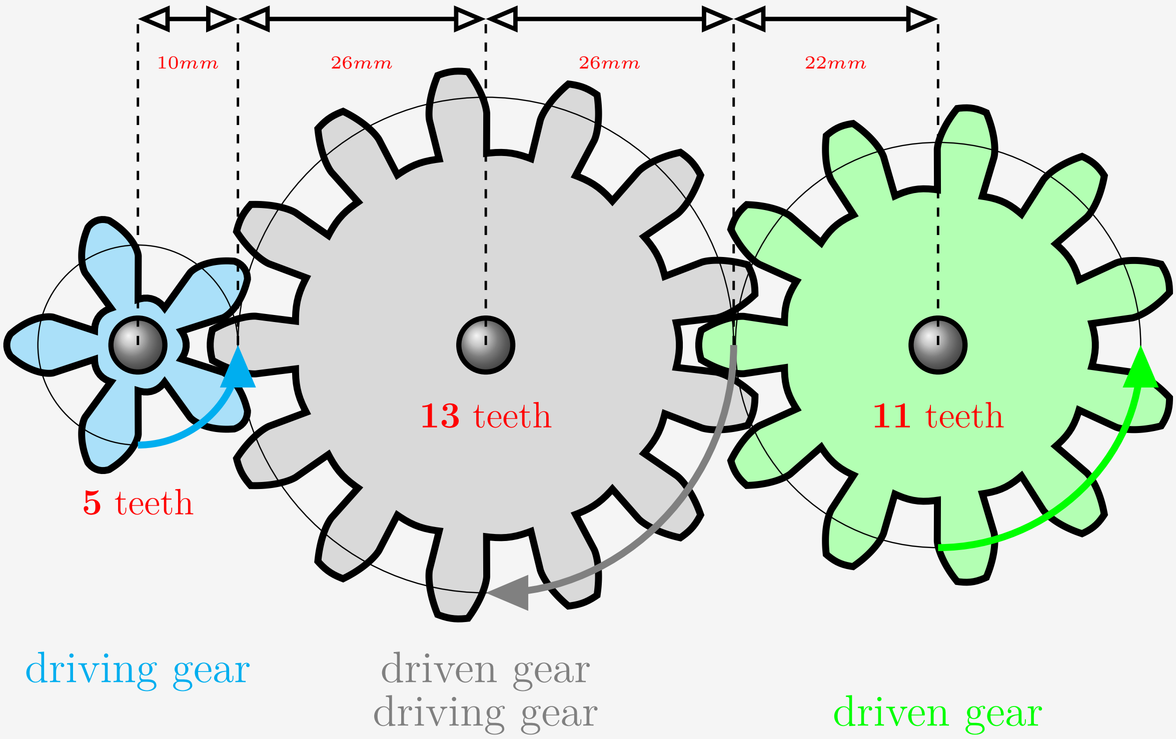

Figure 2: Example of three gears.

Example 3

The next case with four gears is represented in the Plot 4.

The ratios are the following.

$$\frac{19}{9} = 2.\bar{1} \longrightarrow gr = 2.\bar{1} : 1 \quad \text{and} \quad \frac{13}{5} = 2.6 \longrightarrow gr = 2.6 : 1$$

Together, it implies the following end ratio.

$$2.\bar{1} \times 2.6 = 5.4\bar{8} \longrightarrow gr = 5.4\bar{8} : 1$$

With two gears only for the same ratio, it would imply a quite large second gear with ca. 50 teeth.

$$5.4\bar{8} \times 9 \approx 49 \text{ teeth}$$

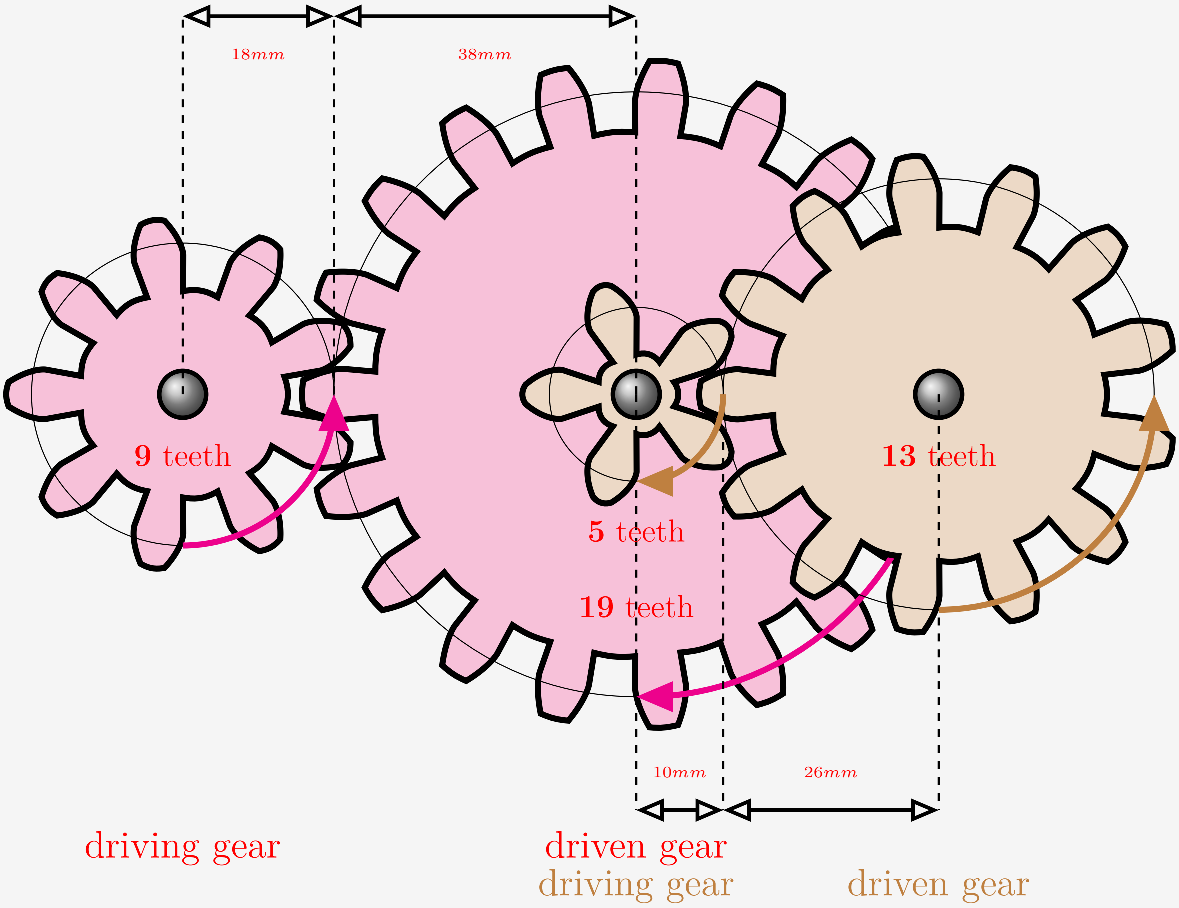

Figure 3: Example of four gears.

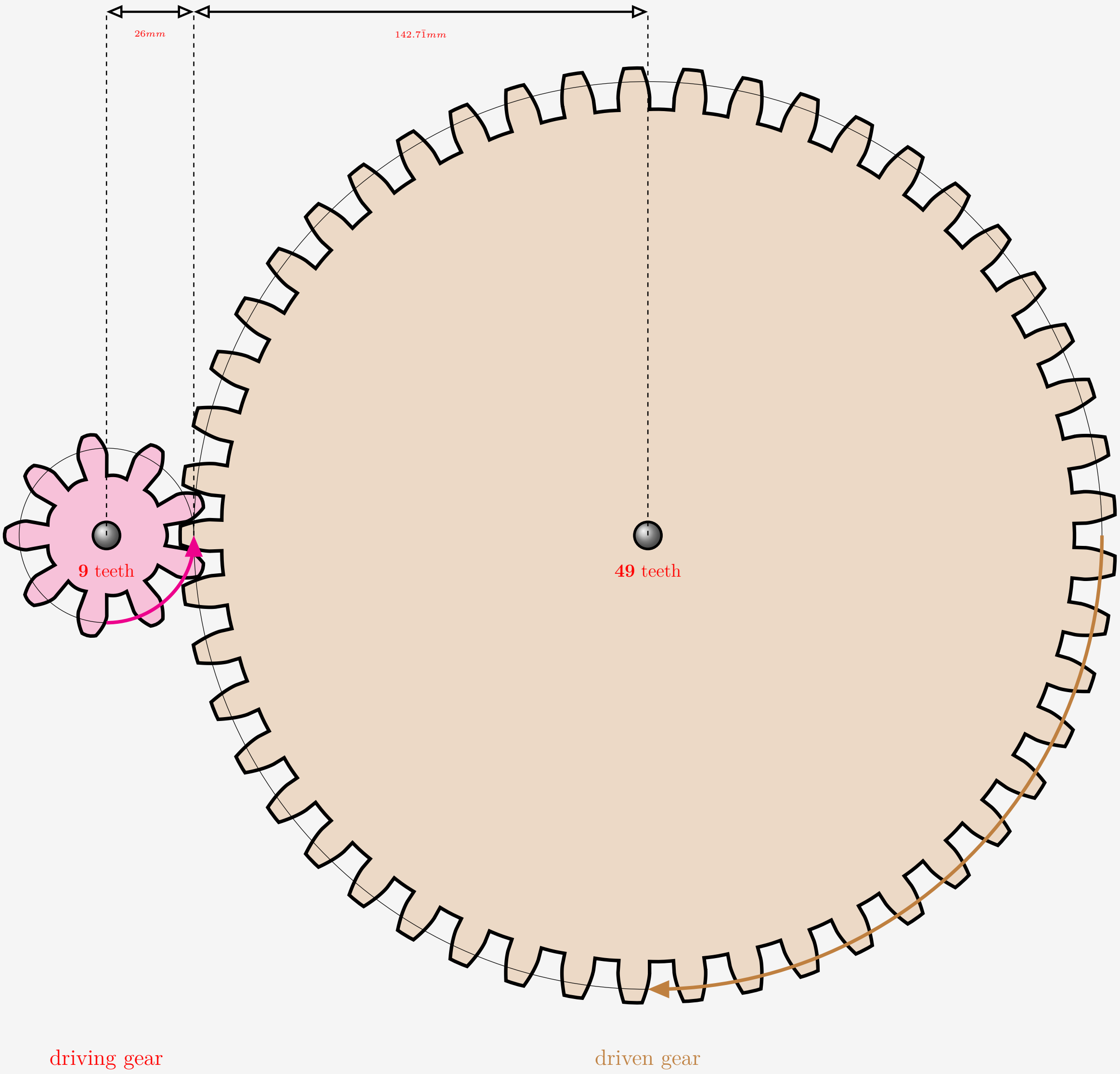

Figure 4: Example of four gears.

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